Formalized Confluence of Quasi-Decreasing, Strongly Deterministic Conditional TRSs*

Christian Sternagel   Thomas Sternagel

University of Innsbruck, Austria

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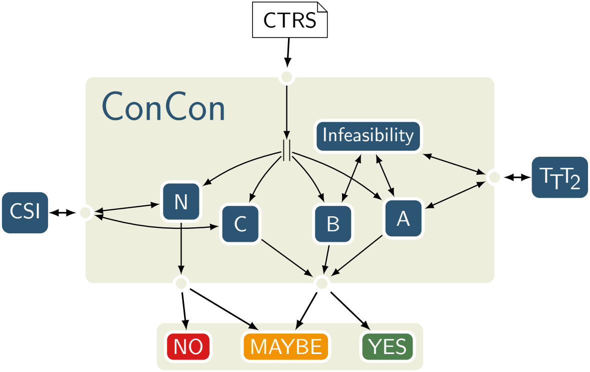
Certification



(A) quasi-decreasing SDTRS $\mathcal{R}$ is confluent $\iff$ all CCPs joinable

(B) almost orthogonal properly oriented right-stable 3-CTRS is confluent

(C) weakly left-linear DCTRS $\mathcal{R}$ is confluent if $U(\mathcal{R})$ is confluent

(N) various heuristics for non-confluence

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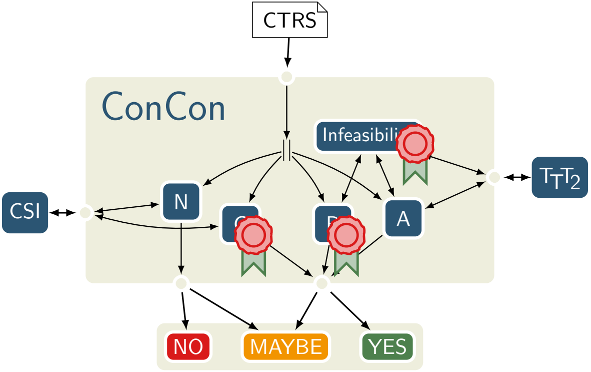

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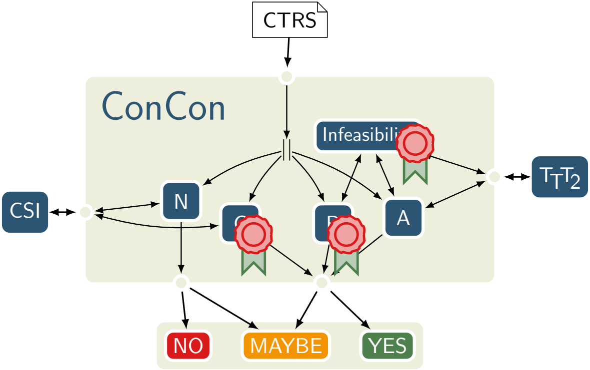

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Literature

On conditional rewrite systems with extra variables and deterministic logic programs
J. Avenhaus, C. Loría-Sáenz
doi: 10.1007/3-540-58216-9_40,
LPAR, 1994.

Theorem

Quasi-reductive SDTRS $\mathcal{R}$ is confluent $\iff$ all CCPs are joinable.

SDTRS

$$\ell \to r \Leftarrow \underbrace{s_1 \approx t_1, \ldots, s_k \approx t_k}_{c}$$

- $\approx$ interpreted as $\to_{\mathcal{R}}^*$
- $\ell \notin \mathcal{V}$
- $\mathcal{V}(r) \subseteq \mathcal{V}(\ell, c)$
- $\mathcal{V}(s_i) \subseteq \mathcal{V}(\ell, t_1, \ldots, t_{i-1})$
- $\forall\sigma$. normalized $\sigma \longrightarrow t_i\sigma \in \mathsf{NF}(\to_{\mathcal{R}})$

Quasi-Decreasingness

SDTRS $\mathcal{R}$ ($\mathcal{F}$) is *quasi-decreasing* if there is $\succ$ on $\mathcal{T}(\mathcal{F},\mathcal{V})$:

- well-founded $\succ$
- $\succ = (\succ \cup \rhd)^+$
- $\rightarrow_{\mathcal{R}} \subseteq \succ$
- $\forall \ell \to r \Leftarrow s_1 \approx t_1, \ldots, s_n \approx t_n \in \mathcal{R},\ \sigma\colon \mathcal{V} \to \mathcal{T}(\mathcal{F},\mathcal{V})$,
  $0 \leqslant i < n$: $\forall 1 \leqslant j \leqslant i.\ s_j\sigma \rightarrow_{\mathcal{R}}^* t_j\sigma \longrightarrow \ell\sigma \succ s_{\mathsf{i+1}}\sigma$

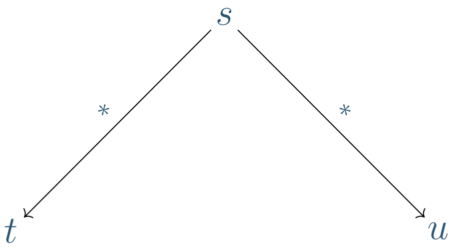
Joinability of CCPs

CCP $u \approx v \Leftarrow c$ is *joinable* if
$\forall \sigma.\ (\forall s \approx t \in c.\ s\sigma \to_{\mathcal{R}}^{*} t\sigma) \longrightarrow u\sigma \downarrow_{\mathcal{R}} v\sigma$

Proof Idea I

SDTRS: $\mathcal{R}$

quasi-decreasing $\mathcal{R}$, all CCPs are joinable $\implies$ confluent $\mathcal{R}$



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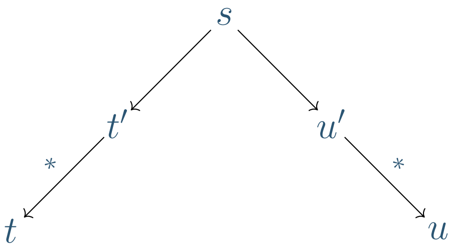

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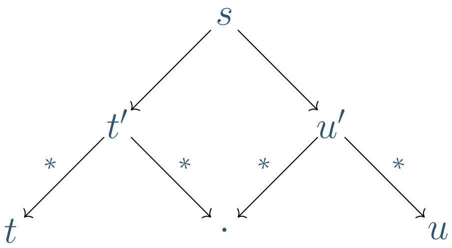

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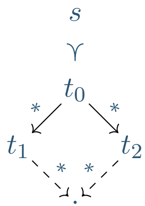
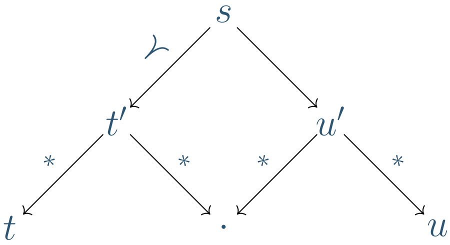
Induction Hypothesis

$\forall t_0, t_1, t_2.$





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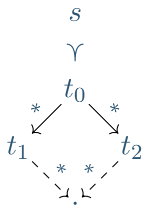

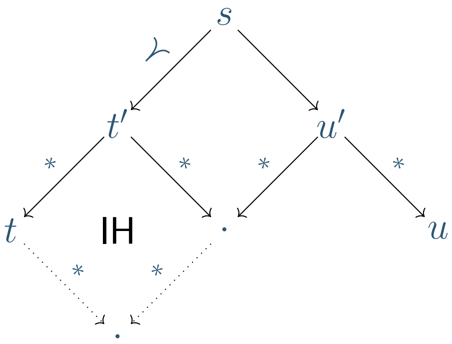

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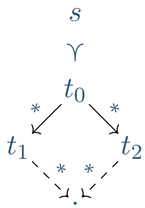

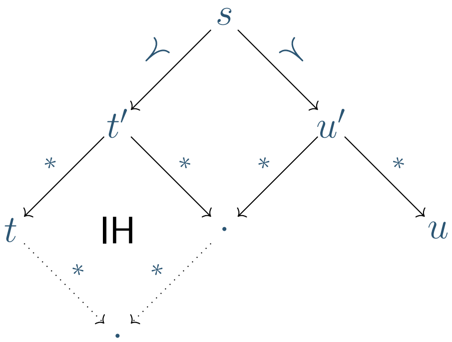

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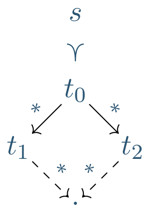

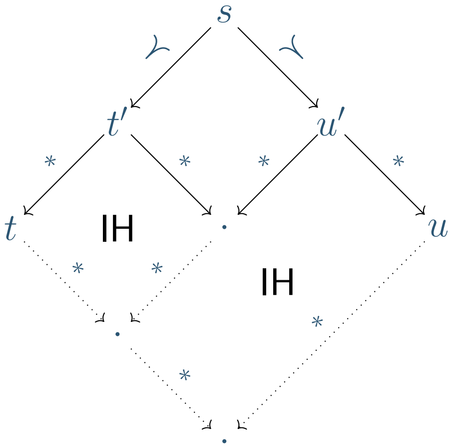

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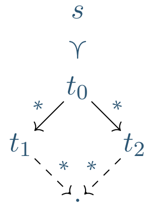

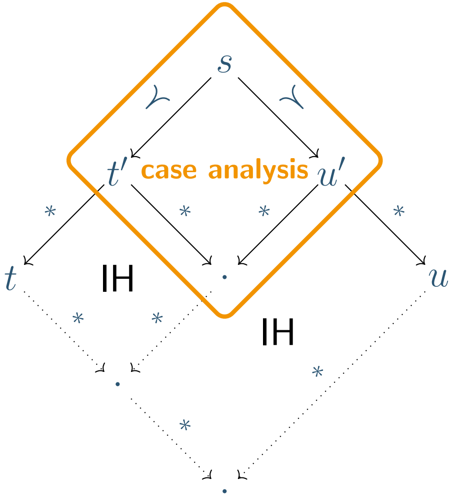

Proof Idea II

$\rho_1 : \ell_1 \to r_1 \Leftarrow c_1,\ \sigma_1,\ p,$
$\forall u \approx v \in c_1.\ u\sigma_1 \to_{\mathcal{R}}^* v\sigma_1$

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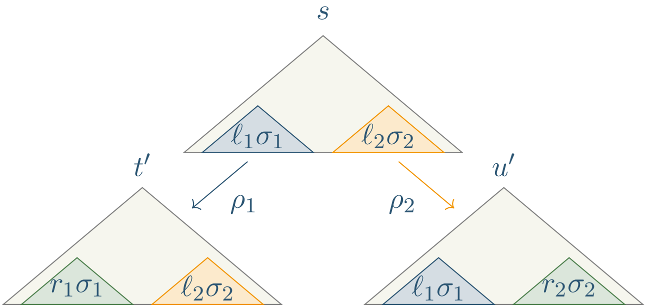
case 1:

$p \,||\, q$



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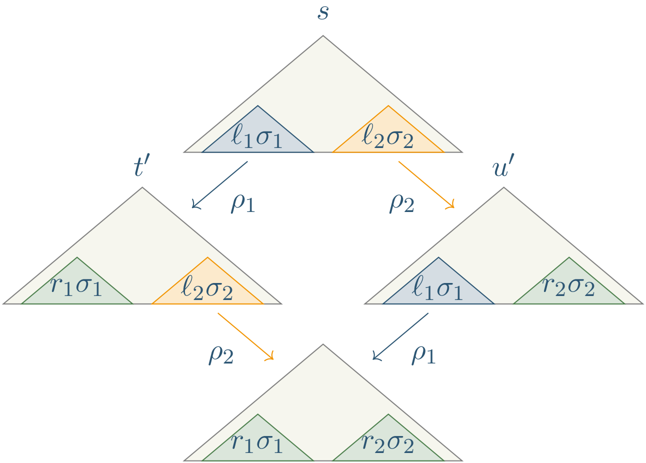

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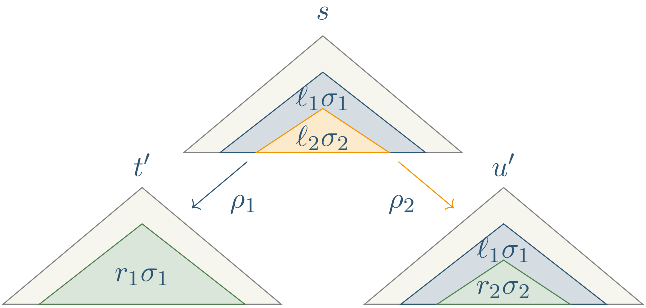

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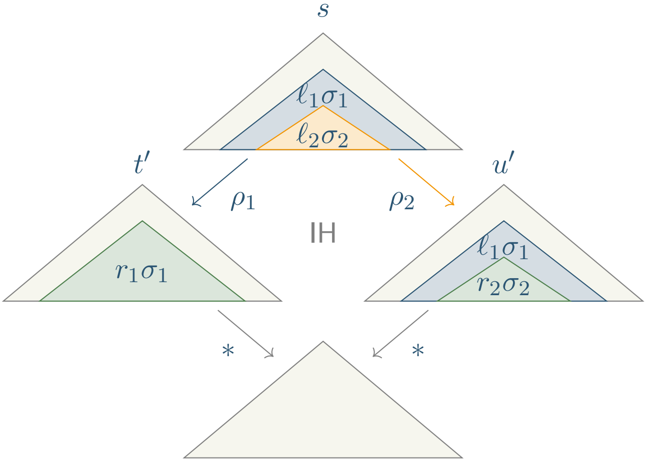

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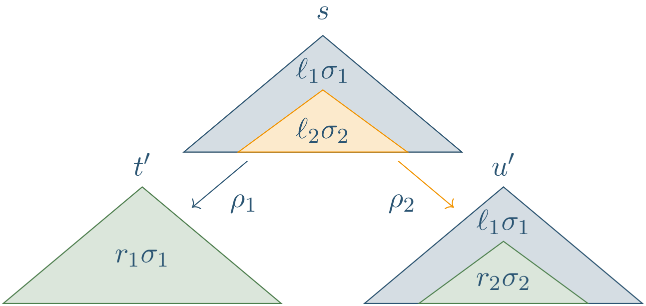
case 3:

$p = \epsilon, p \leqslant q,$
$q \in \mathsf{Pos}_{\mathcal{F}}(\ell_1),$
$q \neq \epsilon \vee \rho_1 \mathrel{\dot{\neq}} \rho_2$



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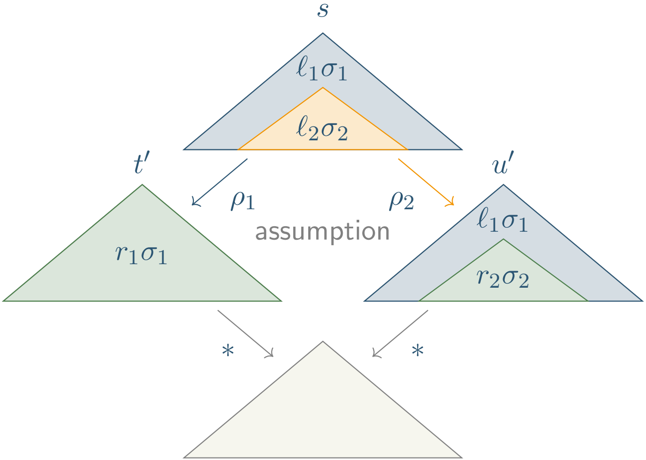

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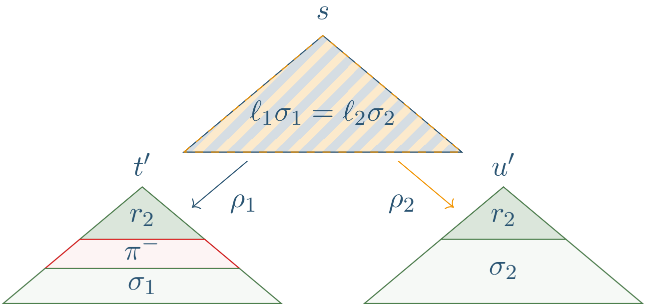
$p = q = \epsilon,$
$\exists \pi.\ \rho_1 \pi = \rho_2$



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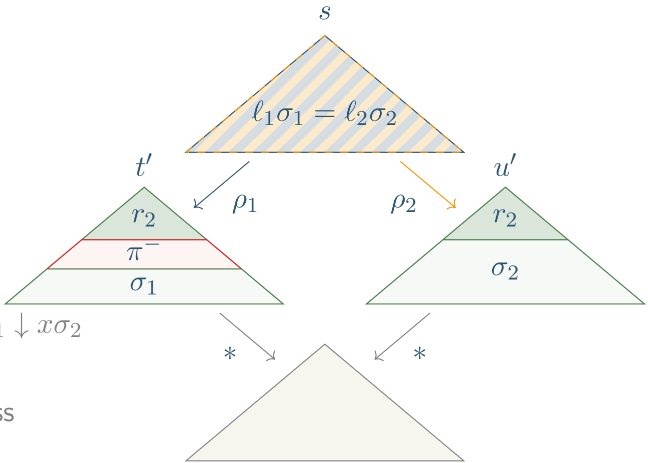

- $\forall x \in \mathcal{V}(\rho_2).\ x\pi^-\sigma_1 \downarrow x\sigma_2$
- induction on $|c_2|$
- quasi-decreasingness
- strong determinism
- IH

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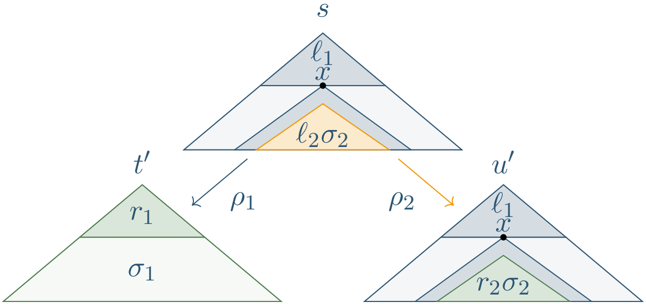

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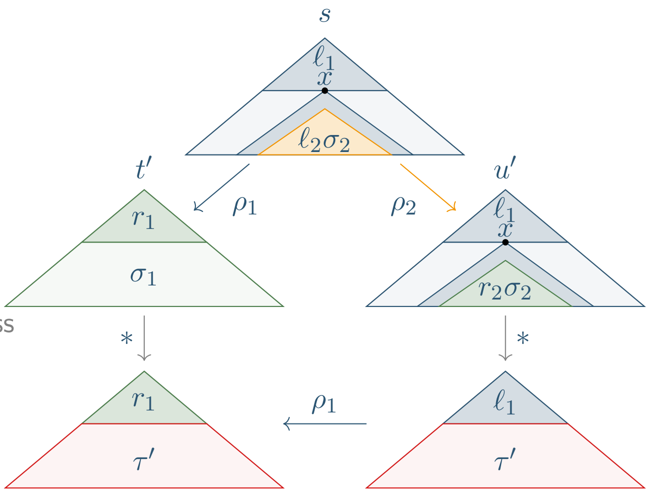

- quasi-decreasingness
- strong determinism
- IH

Conclusion

Remarks

- First version using quasi-reductivity
  - Definition is different from Ohlebusch and IsaFoR
- Using quasi-decreasingness:
  - Straight-forward and made proof easier
  - One lemma from paper not needed anymore
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Challenges

- Paper proof assumes rules to be identical
- Permutations for variable disjoint variants

$t_1 \downarrow t_2$. So assume that this critical pair is improper. Then $t \equiv \sigma_1(l_1) \equiv \sigma_2(l_2)$ and we may assume that $C_1 \Longrightarrow l_1 \to r_1$ and $C_2 \Longrightarrow l_2 \to r_2$ are identical, i.e. $C_i \Longrightarrow l_i \to r_i \equiv C \Longrightarrow l \to r$ for $i = 1, 2$. We have $\sigma_1(x) \equiv \sigma_2(x)$ for all

Appendix

Quasi-Reductivity

Definition 3.1 *Let* $\succ$ *be a reduction ordering on* $\mathcal{T}(\mathcal{F}, \mathcal{V})$. *A DTRS R is* quasi-reductive *wrt.* $\succ$ *if for every substitution* σ *and every rule* $u_1 \to v_1, \ldots, u_n \to v_n \Longrightarrow l \to r$ *in* R

(i) $\sigma(u_j) \succeq \sigma(v_j)$ *for* $1 \leq j \leq i$ *implies* $\sigma(l) \succ_{st} \sigma(u_{i+1})$

(ii) $\sigma(u_j) \succeq \sigma(v_j)$ *for* $1 \leq j \leq n$ *implies* $\sigma(l) \succ \sigma(r)$