Tutorial Intro. to Modern Formal Methods:

Mechanized Formal Analysis
Using Model Checking, Theorem Proving
SMT Solving, Abstraction, and Static Analysis
With SAL, PVS, and Yices, and more

John Rushby

Computer Science Laboratory
SRI International
Menlo Park CA USA

Some Different Approaches to Formal Analysis (of safety properties of concurrent systems defined as transition relations)

. . . to be demonstrated on a concrete example

Namely, Lamport's Bakery Algorithm

- Explicit, symbolic, bounded model checking
- Deduction (theorem proving)
- Abstraction and model checking
- Automated abstraction (failure tolerant theorem proving)
- Bounded model checking (for infinite state systems)

Focus is on pragmatics and tools (many demos), not theory

If there is time and interest, will also look at test generation, static analysis, and timed systems

Formal Methods: Analogy with Engineering Mathematics

- Engineers in traditional disciplines build mathematical models of their designs
- And use calculation to establish that the design, in the context of a modeled environment, satisfies its requirements
- Only useful when mechanized (e.g., CFD)
- Used in the design loop (exploration, debugging)
    - Model, calculate, interpret, repeat
- Also used in certification
    - Verify by calculation that the modeled system satisfies certain requirements
- Need to be sure that model faithfully represents the design, design is implemented correctly, environment is modeled faithfully, and calculations are performed without error

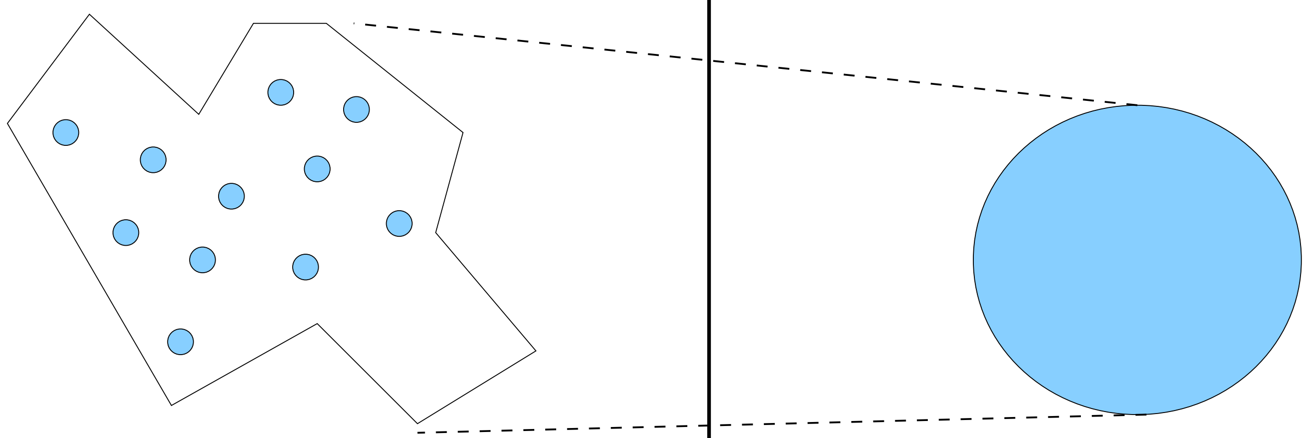
Formal Methods: Analogy with Engineering Math (ctd.)

- Formal methods: same idea, applied to computational systems
- The applied math of Computer Science is formal logic
- So the models are formal descriptions in some logical system
  - E.g., a program reinterpreted as a mathematical formula rather than instructions to a machine
- And calculation is mechanized by automated deduction: theorem proving, model checking, static analysis, etc.
- Formal calculations (can) cover all modeled behaviors
- If the model is accurate, this provides verification
- If the model is approximate, can still be good for debugging (aka. refutation)

Formal Methods: In Pictures

Testing/Simulation

Formal Analysis



Real System

Formal Model

- Partial coverage

- Complete coverage
  (of the modeled system)

  Accurate model: verification

  Approximate model: debugging

Comparison with Simulation, Testing etc.

- Simulation also considers a model of the system (designed for execution rather than analysis)
- Testing considers the real thing
- Both differ from formal methods in that they examine only some of the possible behaviors
- For continuous systems, verification by extrapolation from partial tests is valid, but for discrete systems, it is not
- Can make only statistical projections, and it's expensive
  - 114,000 years on test for 10^{-9}

  Limit to evidence provided by testing is about 10^{-4}
- In most applications, testing is used for debugging rather than verification

Comparison with Simulation, Testing etc. (ctd)

- Debugging depends on choosing right test cases
  - Can be improved by explicit coverage measures
  - Good coverage is almost impossible when the environment can introduce huge numbers of different behaviors (e.g., fault arrivals, real-time, asynchronous interactions)

  So depends on skill, luck
- Since formal methods can consider all behaviors, certain to find the bugs
  - Provided the model, environment, and the properties checked are sufficiently accurate to manifest them

  So depends on skill, luck
- Experience is you find more bugs (and more high-value bugs) by exploring all behaviors of an approximate model than by exploring some behaviors of a more accurate one

Formal Calculations: The Basic Challenge

- Build mathematical model of system and deduce properties by calculation
- The applied math of computer science is formal logic
- So calculation is done by automated deduction
- Where all problems are NP-hard, most are superexponential (2^{2^n}), nonelementary ($2^{2^{2^{\cdot^{\cdot^{\cdot}}}}}\}^n$), or undecidable
- Why? Have to search a massive space of discrete possibilities
- Which exactly mirrors why it's so hard to provide assurance for computational systems
- But at least we've reduced the problem to a previously unsolved problem!

Formal Calculations: Meeting The Basic Challenge

Ways to cope with the massive computational complexity

- Use human guidance
  - That's interactive theorem proving—e.g., PVS
- Restrict attention to specific kinds of problems
  - E.g., model checking—focuses on state machines etc.
- Use approximate models, incomplete search
  - model checkers are often used this way
- Aim at something other than verification
  - E.g., bug finding, test case generation
- Verify weak properties
  - That's what static analysis typically does
- Give up soundness and/or completeness
  - Again, that's what static analysis typically does

Let's do an example: Bakery

The Bakery Algorithm for Distributed Mutual Exclusion

- Idea is based on the way people queue for service in US delicatessens and bakeries (or Vietnam Airlines in HCMC)
- A machine dispenses tickets printed with numbers that increase monotonically
- People who want service take a ticket
- The unserved person with the lowest numbered ticket is served next
  - Safe: at most one person is served (i.e., is in the "critical section") at a time
  - Live: each person is eventually served
- Preserve the idea without centralized ticket dispenser

Informal Protocol Description

- Works for $n \geq 1$ processes
- Each process has a ticket register, initially zero
- When it wants to enter its critical section, a process sets its ticket greater than that of any other process
- Then it waits until its ticket is smaller than that of any other process with a nonzero ticket
- At which point it enters its critical section
- Resets its ticket to zero when it exits its critical section
- Show that at most one process is in its critical section at any time (i.e., mutual exclusion)

Formal Modeling and Analysis

- Build a mathematical model of the protocol
- Analyze it for a desired property
- Must choose how much detail to include in the model
  - Too much detail: analysis may be infeasible
  - Too little detail: analysis may be inaccurate
    (i.e., fail to detect bugs, or report spurious ones)
  - Must also choose a modeling style that supports intended form of analysis
- Requires judgment (skill, luck) to do this

Modeling the Example System and its Properties: Accuracy and Level of Detail

- The protocol uses shared memory and is sensitive to the atomicity of concurrent reads and writes
- And to the memory model (on multiprocessors with relaxed memory models, reads and writes from different processors may be reordered)
- And to any faults the memory may exhibit
- If we wish to examine the mutual exclusion property of a particular implementation of the protocol, we will need to represent the memory model, fault model, and atomicity employed—which will be quite challenging
- Abstractly (or at first), we may prefer to focus on the behavior of the protocol in an ideal environment with fault-free sequentially consistent atomic memory

Modeling the Example System and its Properties (ctd.)

- Also, although the protocol is suitable for n processes, we may prefer to focus on the important special case $n = 2$
- And although each process will perform activities other than the given protocol, we will abstract these details away and assume each process is in one of three phases

  idle: performing work outside its critical section

  trying: to enter its critical section

  critical: performing work inside its critical section

Formalizing the Model (continued)

- We will need to model a system state comprising
  For each process:
  - The value of its ticket, which is a natural number
  - The phase it is in—recorded in its "program counter" which takes values `idle`, `trying`, `critical`
- Then we model the (possibly nondeterministic) transitions in the system state produced by each protocol step
- And check that the desired property is always preserved

A Formal Description of the Protocol (in SAL)

```
bakery : CONTEXT =
BEGIN
   phase : TYPE = {idle, trying, critical};
   ticket: TYPE = NATURAL;

process : MODULE =
BEGIN
 INPUT other_t: ticket
 OUTPUT my_t: ticket
 OUTPUT pc: phase

INITIALIZATION
  pc = idle;
  my_t = 0
```

More Formal Protocol Description (continued)

```
TRANSITION
[
     try: pc = idle -->
          my_t'  = other_t + 1;
          pc' = trying
[]
   enter: pc = trying AND (other_t = 0 OR my_t < other_t) -->
          pc' = critical
[]
  leave:  pc = critical -->
          my_t' = 0;
          pc' = idle
]
END;
```

More Formal Protocol Description (continued again)

```
P1 : MODULE = RENAME pc TO pc1 IN process;

P2 : MODULE = RENAME other_t TO my_t,
                     my_t TO other_t,
                     pc TO pc2 IN process;

system : MODULE = P1 [] P2;

safety: THEOREM
    system |- G(NOT (pc1 = critical AND pc2 = critical));

END
```

Analyzing the Specification Using Model Checking

- They are called model checkers because they check whether a system, interpreted as an automaton, is a (Kripke) model of a property expressed as a temporal logic formula
- The simplest type of model checker is one that does explicit state exploration
    - Basically, a simulator that remembers the states it's seen before and backtracks to explore all of them (either depth-first, breadth-first, or a combination)
    - Defeated by the state explosion problem at around a few tens of millions of states
- To get further, need symbolic representations (a short formula can represent many explicit states)
    - Symbolic model checkers use BDDs
    - Bounded model checkers use SAT and SMT solvers

Explicit State Reachability Analysis

- Keep a set of all states visited so far, and a list of all states whose successors have not yet been calculated
  - Initialize both with the initial states
- Pick a state off the list and calculate all its successors
  - i.e., run all possible one-step simulations from that state

  Throw away those seen before
- Add new ones to the set and the list
- Check each new state for the desired (invariant) properties
  - More complex properties use Büchi automaton in parallel
- Iterate to termination, or some state fails a property
  - Or run out of memory, time, patience
- On failure, counterexample (backtrace) manifests problem

Analyzing the Spec'n Using Explicit State Model Checking

- We'll use `sal-esmc`
  - An explicit-state LTL model checker for SAL
  - Not part of the SAL distribution, just used for demos

  Later, we'll look at symbolic and bounded model checking
- This is an infinite-state specification, so cannot enumerate the whole state space, but `sal-esmc` will do its best...

```
sal-esmc -v 3 bakery safety
building execution engine...
  num. bits used to encode a state: 44
verifying property using depth-first search...
  computing set of initial states...
  number of initial states: 1
  number of visited states: 1001, states to process: 1001, depth: 749
  number of visited states: 10001, states to process: 10001, depth: 7499
  number of visited states: 20001, states to process: 20001, depth: 14999
...
```

Bug Finding Using Explicit State Model Checking

- So verification will take forever
- But can be useful for finding bugs: if we remove the +1 adjustment to the tickets, we get a counterexample

```
INVALID, generating counterexample...
number of visited states: 7,  verification time: 0.02 secs
------------------------
my_t = 0   other_t = 0
pc1 = idle pc2 = idle
------------------------
pc2 = trying
------------------------
pc2 = critical
------------------------
pc1 = trying
------------------------
my_t = 0       other_t = 0
pc1 = critical pc2 = critical
```

Verification by Finite State Model Checking

- For traditional methods of model checking, we need to make the state space finite
- Use property preserving abstractions (later)
- Or drastic simplification (“downscaling”)
  - We’ve already done this to some extent, by fixing the number of processors, n, as 2
  - We also need to set an upper bound on the tickets
- We’ll start at 8, then raise the limit to 80, 800, ... until the search becomes too slow
- We have to modify the protocol to bound the tickets
  - So it’s not the same protocol
  - May miss some bugs, or get spurious ones
  - But it’s a useful check

The Bounded Specification in SAL

```
bakery : CONTEXT =
BEGIN
   phase : TYPE = {idle, trying, critical};
   max: NATURAL = 8;
   ticket: TYPE = [0..max];

process : MODULE =
BEGIN
 INPUT other_t: ticket
 OUTPUT my_t: ticket
 OUTPUT pc: phase

INITIALIZATION
  pc = idle;
  my_t = 0
```

The Bounded Specification in SAL (continued)

```
TRANSITION
[
     try: pc = idle AND other_t < max -->
          my_t'  = other_t + 1;
          pc' = trying
[]
   enter: pc = trying AND (other_t = 0 OR my_t < other_t) -->
          pc' = critical
[]
  leave: pc = critical -->
          my_t' = 0;
          pc' = idle
]
END;
```

The Bounded Specification in SAL (continued again)

```
P1 : MODULE = RENAME pc TO pc1 IN process;

P2 : MODULE = RENAME other_t TO my_t,
                     my_t TO other_t,
                     pc TO pc2 IN process;

system : MODULE = P1 [] P2;

safety: THEOREM
    system |- G(NOT (pc1 = critical AND pc2 = critical));

END
```

Results of Model Checking

- For `max = 800`, `sal-esmc` reports

```
sal-esmc -v 3 smallbakery safety
  num. bits used to encode a state: 24
verifying property using depth-first search...
  computing set of initial states...
  number of initial states: 1
  number of visited states: 1001, states to process: 1001, depth: 749
  number of visited states: 2001, states to process: 2001, depth: 1499
  number of visited states: 3001, states to process: 3001, depth: 2249
  number of visited states: 4002, states to process: 804, depth: 602
  number of visited states: 5002, states to process: 1804, depth: 1352
  number of visited states: 6002, states to process: 2804, depth: 2102
  number of visited states: 6397
  verification time: 0.54 secs
proved.
```

- For `max = 8000`, number of visited states grows to 63,997, and time to 23.86 secs

More Explicit State Checks

- Sometimes properties are true for the wrong reason
- It is prudent to introduce a bug and make sure it is detected before declaring victory
  - We can remove the +1 adjustment to tickets and get a counterexample as before
- We can check that the counters are capable of increasing indefinitely by adding the invariant
  `unbounded: THEOREM system |- G(my_t < max);`
  (After undoing the deliberate errors just introduced)
- We get another counterexample
- The pattern is: `P1 tries`, then the following sequence repeats
  `P1 enters`, `P2 tries`, `P1 quits`, `P2 enters`, `P1 tries`, `P2 quits`

Benefits of Explicit State Model Checking

- Can only explore a few million states, but that's enough when there are plenty of bugs to find
- Can use hashing (supertrace) to go further
- Can have arbitrarily complex transition relation
  - Language can include any datatypes and operations supported by the API
- Breadth first search finds short counterexamples
  - Can write special search strategies to target specific cases, or to ignore others (symmetry, partial order)
- LTL is handled via Büchi automata
- Can evaluate functions, not just predicates, on the reachable states: can calculate worst-cases, do optimization

But it runs out of steam on big examples

Analysis by Symbolic Model Checking (SMC)

- We could take a symbolic representation of the transition relation and repeatedly compose it with itself until it reaches a fixpoint (must exist for finite systems)
- That would give us a representation of the reachable states, from which we could check safety properties directly
  - Again, LTL is handled via Büchi automata
- Reduced Ordered Binary Decision Diagrams (BDDs) are a suitable representation for doing this
- This is the basic idea of symbolic model checking
  - In practice, use different methods of calculation, depending on the type of property being checked
- Can construct counterexamples for false properties
- Can (sometimes) handle huge statespaces very efficiently

Symbolic Model Checking (ctd)

- Our symbolic representation is a purely Boolean one
- So we have to compile the transition relation down to what is essentially a circuit
  - Bounded integers represented by bitvectors of suitable size
  - Addition requires the circuit for a boolean adder
  - Similarly for other datatypes and operations
- We are doing a kind of circuit synthesis
- May fail for large or complex transition relations
- BDD operations depend on finding a good variable ordering
- Automated heuristics usually effective to about 600 state bits
- After that, manual ordering, special tricks are needed
- Seldom get beyond 1,000 state bits

Symbolic Model Checking with SAL

- We'll use `sal-smc`, uses CUDD for its BDD operations
- For `max = 80`

  `sal-smc smallbakery safety`

  Requires 46 BDD variables, 240 iterations, 637 states, 2.9 seconds to get to proved
- For `max = 8000`, it'd be tedious doing as above, but

  `sal-smc smallbakery safety --backward`

  Requires 70 BDD variables, 5 iterations, 131,180,871 states, 0.5 seconds to get to `proved`
- Symbolic model checking is "automatic"
- But requires dial twiddling

More Checks With Symbolic Model Checking

- Do the tickets always increase without bound?
  `bounded: LEMMA system |- F(my_t > 3);`
- The counterexample to this liveness property is a prefix, followed by a loop (lasso)
- The pattern is P1 tries, enters, leaves, and then repeats
- Does P1 ever get into the critical region?
  `liveness: THEOREM system |- F(pc1 = critical);`
  Same counterexample as above (with `P2` rather than `P1`)
- If it tries, does it always succeed?

```
response: THEOREM system |-
   G(pc1 = trying => F(pc1 = critical));
Proved
```

Yet More Checks With Symbolic Model Checking

- If it tries infinitely often, does it eventually succeed infinitely often?

```
weak_fair: THEOREM system |-
   F(G(pc1 = trying)) => G(F(pc1 = critical))
Proved
```

- If it tries continuously, does it eventually succeed infinitely often?

```
strong_fair: THEOREM system |-
   G(F(pc1 = trying)) => G(F(pc1 = critical))
Proved
```

- These properties are getting complicated
- We should look at LTL

Linear Temporal Logic

- A language for specifying properties of the execution traces of a system
- Given a system specified by initiality predicate I and transition relation T, a trace is an infinite sequence of states $s = s_0, s_1, \ldots, s_i, \ldots$ where $I(s_0)$ and $T(s_i, s_{i+1})$
- The semantics of LTL defines whether a trace s satisfies a formula p (written as $s \models p$)
- The base cases are when p is a predicate on states, and the operators X (next), and U (strong until)
- $s \models \phi$, where ϕ is a predicate on states, iff ϕ is true on the initial state, i.e., $\phi(s_0)$
- $s \models X(p)$ iff $w \models p$ where $w = s_1, \ldots, s_i, \ldots$
- $s \models U(p, q)$ iff $\exists n : s = s_0, s_1, \ldots s_n.w$,
  $\forall i \in \{0, ..., n\} : s_i, \ldots s_n.w \models p$, and $w \models q$

Linear Temporal Logic (ctd)

- R (release), G (always), F (eventually), B (before), and W (weak until) are defined in terms of these

```
R(p, q) = NOT U(NOT p, NOT q)
G(p) = R(FALSE, p)
F(p) = U(TRUE, p)
B(p, q) = R(p, NOT q)
W(p, q) = G(p) OR U(p, q)
```

- Iterated next state can be defined in SAL

```
XXXX(a:boolean): boolean = X(X(X(X(a))));
```

Or even

```
posnat: TYPE = {x: natural | x>0};
Xn(a: boolean, n: posnat): boolean =
   IF n = 1 THEN X(a) ELSE Xn(X(a), n-1) ENDIF;
X24(a: boolean): boolean = Xn(a, 24);
```

Fairness etc.

- `G(F(expr))`: `expr` is true infinitely often
- `G(F(NOT en OR oc))`: if `en` is enabled continuously, then `oc` will occur infinitely often
  - Weak fairness, often sufficient for progress in asynchronous systems
  - Easier as `G(F(NOT en) OR F(oc))` or `G(G(en) => F(oc))`
- `G(F(en)) => G(F(oc))`: if `en` is enabled infinitely often, then `oc` will occur infinitely often
  - Strong fairness, often necessary for synchronous interaction
- `G(en => F(oc))`: everytime `en` is true, eventually `oc` will also be true; this is a response formula
- `init => G(expr)`: `expr` is always true in any trace that begins with a state satisfying `init`

Complex LTL Formulas

- Can vizualize complex LTL formulas as Büchi automata

  `ltl2buchi smallbakery response -dotty`
  dotty is broken in Ubuntu 8.04, so we have to do
  `ltl2buchi smallbakery response | neato -tps | gv -`
- There are web pages giving LTL for common requirements
- Other property languages
  - CTL: computation tree logic is a branching time logic
    - ⋆ LTL and CTL are incomparable
    - ⋆ SAL accepts CTL syntax on the common fragment

    PSL: Accellera Property Specification Language is a language developed by industry
- For safety properties, may prefer a synchronous observer
  - Module that observes the system, sets error flag when it sees a violation
  - Model check for `G(NOT error)`

From Symbolic to Bounded Model Checking

- Using a different example, `sal-smc -v 3 om1 validity`
  - Oral Messages algorithm with n “relays”
- With 3 relays, 10,749,517,287 reachable states
- With 4 relays, 66,708,834,289,920 reachable states
- With 5 relays, run out of patience waiting for counterexample
- Bounded model checkers are specialized to finding counterexamples
- Sometimes can handle bigger problems than SMC

Analysis by Bounded Model Checking (BMC)

- Given system specified by initiality predicate I and transition relation T on states S
- Is there a counterexample to property P in k steps or less?
- Can try $k = 1, 2, \ldots$
- Find assignment to states $s_0, \ldots, s_k$ satisfying

  $I(s_0) \wedge T(s_0, s_1) \wedge T(s_1, s_2) \wedge \cdots \wedge T(s_{k-1}, s_k) \wedge \neg(P(s_1) \wedge \cdots \wedge P(s_k))$
- Given a Boolean encoding of I, T, and P (i.e., circuit), this is a propositional satisfiability (SAT) problem
- SAT solvers have become amazingly fast recently (see later)
- BMC uses same front end reduction to a Boolean representation as SMC, but a different back end
- BMC generally needs less tinkering than SMC

Bounded Model Checking with SAL

- We'll use `sal-bmc`, Yices as its SAT solver (can use many others); the depth k defaults to 10
- Finds the counterexample in the OM1 example in a few seconds
  `sal-bmc -v 3 om1 validity -d 3`
- And also all these examples
  `sal-bmc -v 3 smallbakery liveness`
  `sal-bmc -v 3 smallbakery bounded`
  `sal-bmc -v 3 smallbakery unbounded -d 20`
- But what about the true property?
  `sal-bmc -v 3 smallbakery safety -d 20`
  Can keep increasing the depth, but what does that tell us?

Extending BMC to Verification

- We should require that $s_0, \ldots, s_k$a are distinct
  - Otherwise there's a shorter counterexample
- And we should not allow any but s_0 to satisfy I
  - Otherwise there's a shorter counterexample
- If there's no path of length k satisfying these two constraints, and no counterexample has been found of length less than k, then we have verified P
  - By finding its finite diameter
- Seldom feasible in practice

Alternatively, Automated Induction via BMC

- Ordinary inductive invariance (for P):

  Basis: $I(s_0) \supset P(s_0)$

  Step: $P(r_0) \wedge T(r_0, r_1) \supset P(r_1)$

- Extend to induction of depth k:

  Basis: No counterexample of length k or less

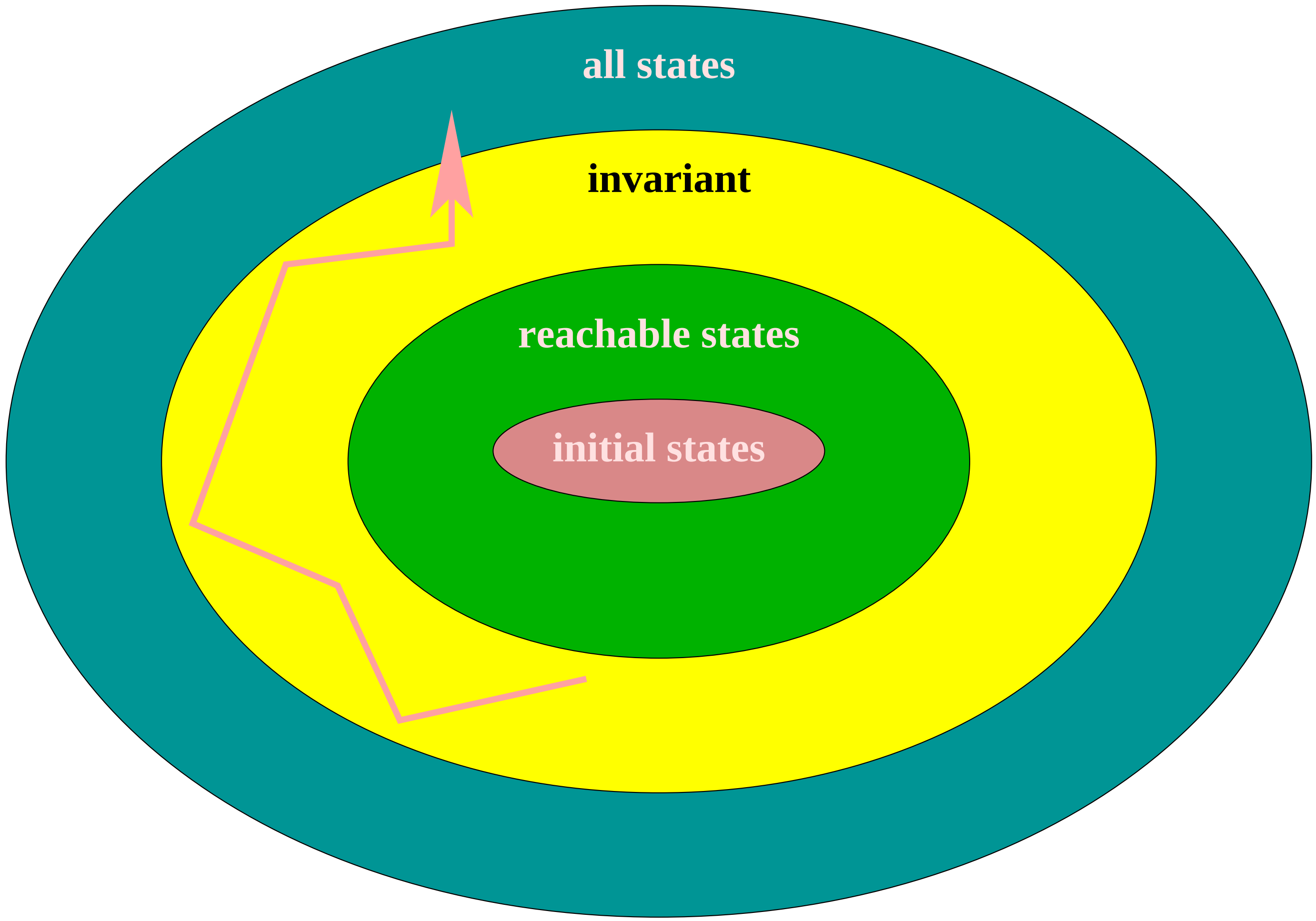
  Step: $P(r_0) \wedge T(r_0, r_1) \wedge P(r_1) \wedge \cdots \wedge P(r_{k-1}) \wedge T(r_{k-1}, r_k) \supset P(r_k)$

  These are close relatives of the BMC formulas

- Induction for $k = 2, 3, 4 \ldots$ may succeed where $k = 1$ does not
- Note that counterexamples help debug invariant
- Can easily extend to use lemmas

k-Induction is Powerful

Violations get harder as k grows



Verification by k-Induction

- Looking at an inductive counterexample can help suggest lemmas (idea is to make the initial state infeasible)
  `sal-bmc -v 3 -d 2 -i smallbakery safety -ice`
- Here's a simple lemma

```
aux: LEMMA system |- G((my_t = 0 => pc1 = idle)
                       AND (other_t = 0 => pc2 = idle));
```

- Can prove with `sal-smc`, or with 1-induction

```
sal-bmc -v 3 -d 1 -i smallbakery aux
```

- Can then verify `safety` with 2-induction using this lemma

```
sal-bmc -v 3 -d 2 -i smallbakery safety -l aux
```

Aside: BMC Can Also Solve Sudoku

Bounded Model Checking for Infinite State Systems

- We can discharge the BMC and k-induction efficiently for Boolean encodings of finite state systems because SAT solvers do efficient search
- If we could discharge these formulas over richer theories, we could do BMC and k-induction for state machines over these theories
- So how about if we combine a SAT solver with decision procedures for useful theories like arithmetic?
- That's what an SMT solver does (details later)
  - Satisfiability Modulo Theories
- BMC using an SMT solver yields an infinite bounded model checker
  - i.e., a bounded model checker for infinite state systems

SMT Solvers

- Ours is called Yices
  - Typically does very well in the annual SMT competition
- Yices decides formulas in the combined theories of linear arithmetic over integers and reals (including mixed forms), fixed size bitvectors, equality with uninterpreted functions, recursive datatypes (such as lists and trees), extensional arrays, dependently typed tuples and records of all these, lambda expressions, and some quantified formulas
- Decides whether formulas are unsatisfiable or satisfiable; in the latter case it can construct an explicit satisfying instance
- For unsatisfiable formulas, it can optionally find an assignment that maximizes the weight of satisfied clauses (i.e., MaxSMT) or, dually, find a minimal set of unsatisfiable clauses (the unsat core)

Infinite Bounded Model Checking with SAL

- We'll use `sal-inf-bmc`
- Can repeat the examples we did with BMC using the original specification
- ```
  sal-inf-bmc -v 3 bakery bounded -d 3
  sal-inf-bmc -v 3 bakery liveness
  sal-inf-bmc -v 3 bakery liveness -it
  sal-inf-bmc -v 3 bakery unbounded -d 20
  ```

  ```
  sal-inf-bmc -v 3 -d 1 -i bakery aux
  sal-inf-bmc -v 3 -d 2 -i bakery safety -l aux
  ```
- Infinite BMC and k-induction blur the line between model checking and theorem proving
  ```

Analyzing the Specification Using Theorem Proving

- We'll use PVS
- PVS is a logic, it does not have a notion of state, nor of concurrent programs, built in—we must specify the program using the transition relation semantics of SAL

```
bakery: THEORY
BEGIN
  phase : TYPE = {idle, trying, critical}

  state: TYPE = [# pc1, pc2: phase, t1, t2: nat #]

  s, pre, post: VAR state
```

The Transitions in PVS

```
P1_transition(pre, post): bool =
 IF pre`pc1 = idle
   THEN post = pre WITH [(t1) := pre`t2 + 1, (pc1) := trying]
 ELSIF pre`pc1 = trying AND (pre`t2 = 0 OR pre`t1 < pre`t2)
   THEN post = pre WITH [(pc1) := critical]
 ELSIF pre`pc1 = critical
   THEN post = pre WITH [(t1) := 0, (pc1) := idle]
 ELSE post = pre
 ENDIF

P2_transition(pre, post): bool = ... similar

transitions(pre, post): bool =
  P1_transition(pre, post) OR P2_transition(pre, post)
```

Initialization and Invariant in PVS

```
init(s): bool = s`pc1 = idle AND s`pc2 = idle
                  AND s`t1 = 0 AND s`t2 = 0

safe(s): bool = NOT(s`pc1 = critical AND s`pc2 = critical)

% To prove that a property P is an invariant, we prove it is *inductive*
% This is similar to Amir Pnueli's rule for Universal Invariance
% Except we strengthen the actual property rather than have an auxiliary

indinv(inv: pred[state]): bool =
 FORALL s: init(s) => inv(s)
  AND FORALL pre, post:
     inv(pre) AND transitions(pre, post) => inv(post)

first_try: LEMMA indinv(safe)
```

First Attempted Proof: Step 1

- Starting the PVS theorem prover gives us this sequent

```
first_try :
  |-------
{1}   indinv(safe)
Rule?
```

- The proof commands `(EXPAND "indinv")` and `(GROUND)` open up the definition of `invariant` and split it into cases

- We are then presented with the first of the two cases

```
This yields  2 subgoals:
first_try.1 :
  |-------
{1}   FORALL s: init(s) => safe(s)
```

- This is discharged by the proof command `(GRIND)`, which expands definitions and performs obvious deductions

First Attempted Proof: Step 2

- This completes the proof of first_try.1.

```
first_try.2 :
  |-------
{1} FORALL pre, post:
    safe(pre) AND transitions(pre, post) => safe(post)
```

- The commands (SKOSIMP), (EXPAND "transitions"), and (GROUND) eliminate the quantification and split transitions into separate cases for processes 1 and 2

```
first_try.2.1 :
{-1}  P1_transition(pre!1, post!1)
[-2]  safe(pre!1)
  |-------
[1]   safe(post!1)
```

- (EXPAND "P1_transition") and (BDDSIMP) split the proof into four cases according to the kind of step made by the process

First Attempted Proof: Step 3

- The first one is discharged by (GRIND), but the second is not and we are presented with the sequent

```
first_try.2.1.2 :
[-1]  pre!1`t2 = 0
{-2}  trying?(pre!1`pc1)
[-3]  post!1 = pre!1 WITH [(pc1) := critical]
[-4]  safe(pre!1)
{-5}  critical?(pre!1`pc2)
  |-------
```

- When there are no formulas below the line, a sequent is true if there is a contradiction among those above the line
- Here, Process 1 enters its critical section because Process 2's ticket is zero—but Process 2 is already in its critical section

First Attempted Proof: Aha!

- This is a contradiction because Process 2 must have incremented its ticket (making it nonzero) when it entered its `trying` phase
- But contemplation, or experimentation with the prover, should convince you that this fact is not provable from the information provided
- Similarly for the other unprovable subgoals
- The problem is not that `safe` is untrue, but that it is not inductive
  - It does not provide a strong enough antecedent to support the proof of its own invariance

Second Attempted Proof

- The solution is to prove a stronger property than the one we are really interested in

```
strong_safe(s): bool = safe(s)
  AND (s`t1 = 0 => s`pc1 = idle)
  AND (s`t2 = 0 => s`pc2 = idle)

second_try: LEMMA indinv(strong_safe)
```

Second Attempted Proof: Aha! Again

- The stronger formula deals with the case we just examined, and the symmetric case for Process 2, but still has two unproved subgoals; here's one of them

```
second_try.2 :

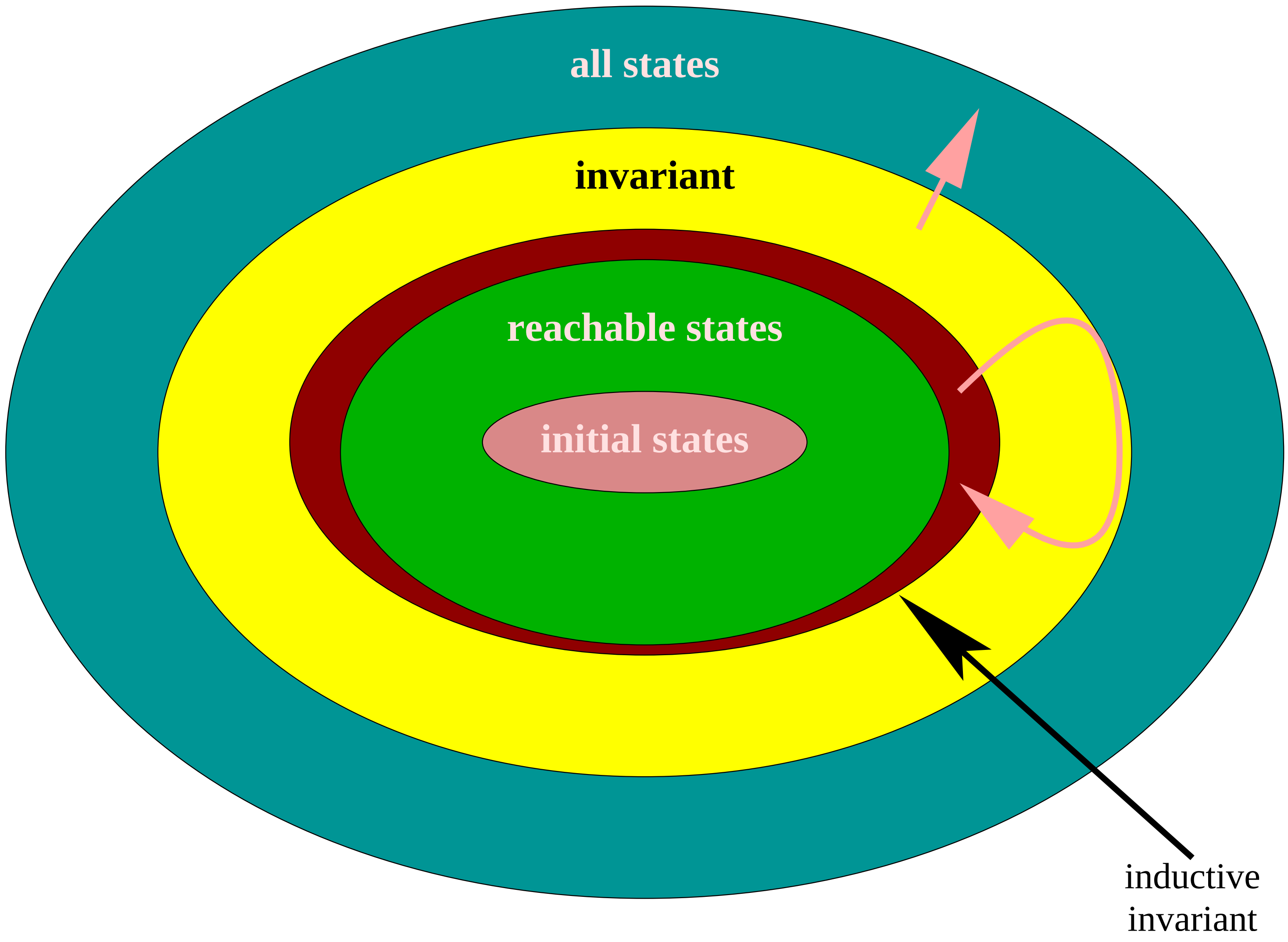
{-1}  trying?(pre!1`pc1)
{-2}  pre!1`t1 < pre!1`t2
{-3}  post!1 = pre!1 WITH [(pc1) := critical]
{-4}  critical?(pre!1`pc2)
  |-------
{1}   (pre!1`t1 = 0)
```

- The situation here is that Process 1 has a smaller ticket than Process 2 and is entering its critical section—even though Process 2 is already in its critical section
- But this cannot happen because Process 2 must have had the smaller ticket when it entered (because Process 1 has a nonzero ticket), contradicting formula -2

Third Attempted Proof

- Again we need to strengthen the invariant to carry along this fact
- ```
  inductive_safe(s):bool = strong_safe(s)
   AND ((s`pc1 = critical AND s`pc2 = trying) => s`t1 < s`t2)
   AND ((s`pc2 = critical AND s`pc1 = trying) => s`t1 > s`t2)

  third_try: LEMMA indinv(inductive_safe)
  ```
- Finally, we have a invariant that is inductive—and is proved with `(GRIND)`
  ```

Inductive Invariants

- To establish an invariant or safety property S (one true of all reachable states) by theorem proving, we invent another property P that implies S and that is inductive (on transition relation T, with initial states I)
  - Includes all the initial states: $I(s) \supset P(s)$
  - Is closed on the transitions: $P(s) \wedge T(s,t) \supset P(t)$
- The reachable states are the smallest set that is inductive, so inductive properties are invariants
- Trouble is, naturally stated invariants are seldom inductive
  - The second condition is violated
- Need to make them smaller (stronger) to exclude the states that take you outside the invariant

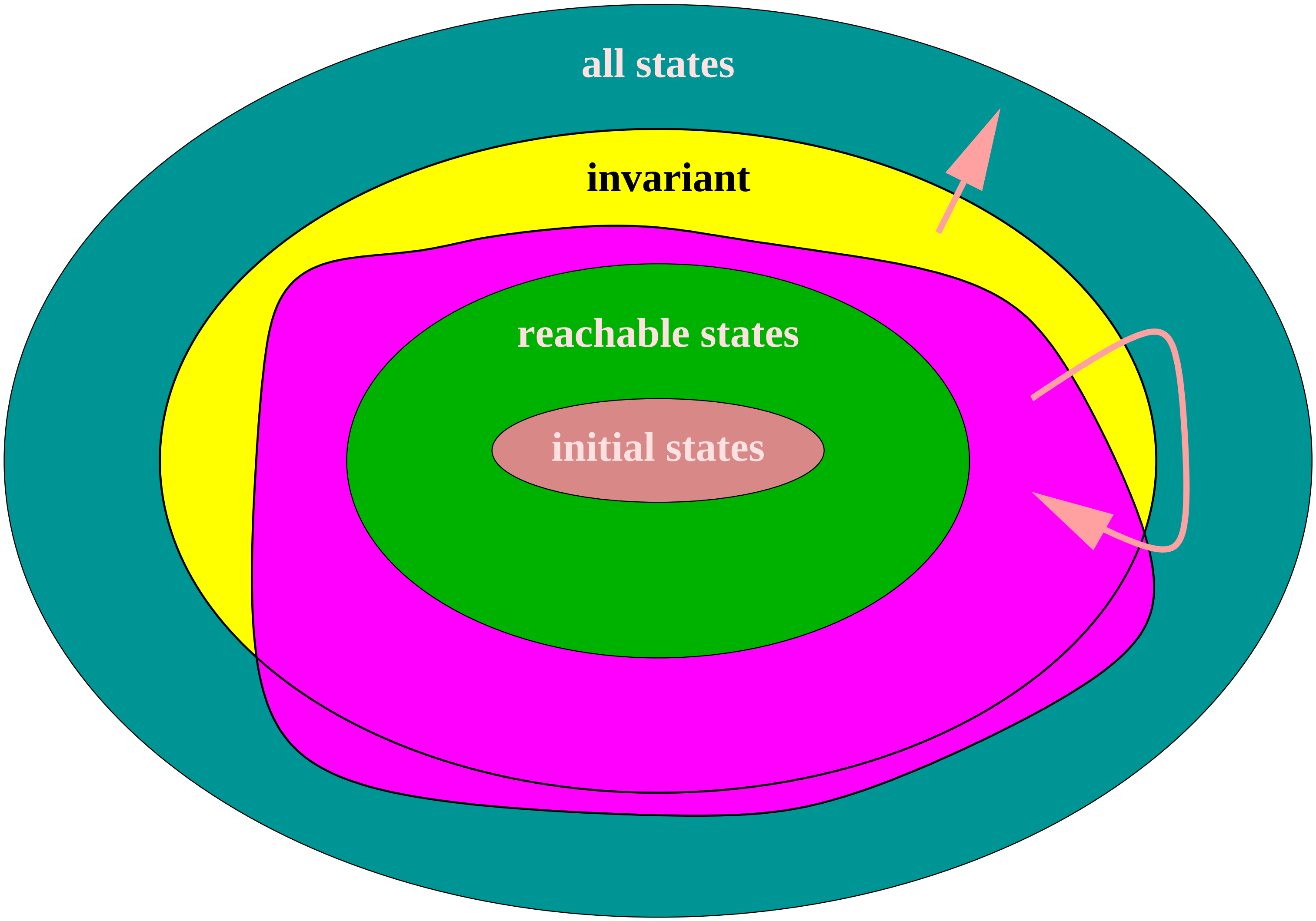
Noninductive Invariants In Pictures



Strengthening Invariants To Make Them Inductive

- Postulate a new invariant that excludes the states (so far discovered) that take you outside the desired invariant
- Show that the conjunction of the new and the desired invariant is inductive
- Iterate until success or exasperation

Inductive Invariants In Pictures



Strengthening Invariants To Make Them Inductive

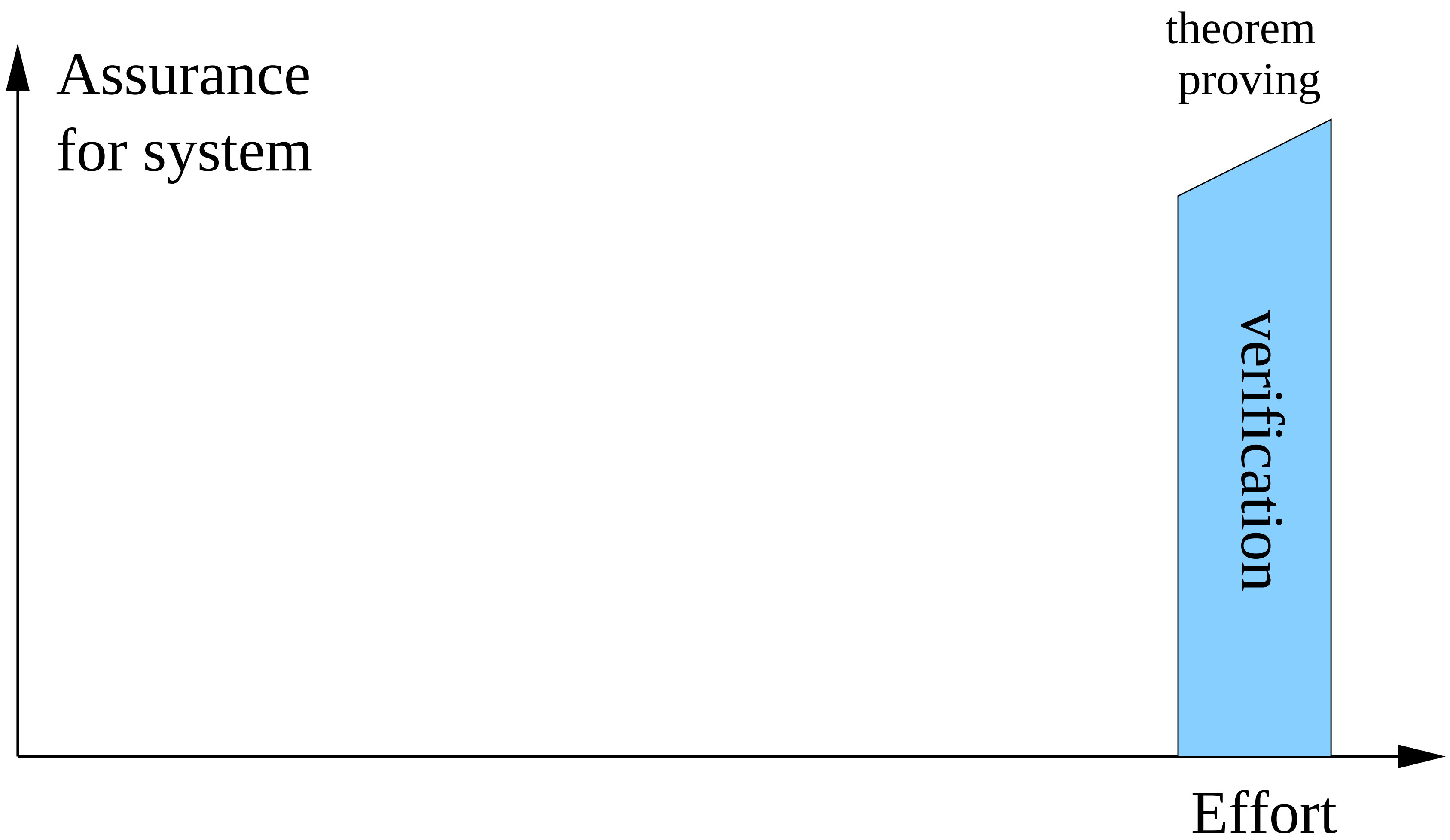
- Iterate until success or exasperation
- Process can be made systematic
  - Each strengthening was suggested by a failed proof

  But is always tedious
- Bounded retransmission protocol required 57 such iterations
  - Took a couple of months to complete
    (Havelund and Shankar)
- Notice that each conjunct must itself be an invariant
  (the very property we are trying to establish)
- Disjunctive invariants are an alternative for some problems
  (see my CAV 2000 paper)

Pros and Cons of Theorem Proving

- Theorem proving can handle infinite state systems
- And accurate models
  - Sometimes less says more–e.g., fault tolerance
- And general properties
  (not just those expressible in temporal logic)
- But it's hard (and not everyone finds it fun)
  - Everything is possible but nothing is easy
  - Especially strengthening of invariants
- Interaction focuses on proof, and idiosyncrasies of the prover, not on the design being evaluated
  - "Interactive theorem proving is a waste of human talent"
- It's all or nothing
  - No incremental return for incremental effort

Formal Verification by Theorem Proving: The Wall



Aside: Design Choices in PVS

- Aside from the need to strengthen the invariant, PVS did OK on this example (and does so on many more)
- We only used a fraction of its linguistic resources
  - Higher-order logic with dependent predicate subtyping
  - Recursive abstract data types and inductive types
  - Parameterized theories and interpretations
  - . . . and most of it is efficiently executable
- It automatically discharged subgoals by deciding properties over abstract data types (enumeration types are a degenerate case), integer arithmetic, record updates, prop'nl calculus
- In larger examples, it also has to choose when to open up a definition, and when to apply a rewrite
- What makes PVS (and other verification systems) effective is that it has tightly integrated automation for all of these

Top-Level Design Choices in PVS

- Specification language is a higher-order logic with subtyping
  - Typechecking is undecidable: uses theorem proving
- User supplies top-level strategic guidance to the prover
  - Invoking appropriate proof methods (induction etc.)
  - Identifying necessary lemmas
  - Suggesting case-splits
  - Recovering when automation fails
- Automation takes care of the details, through a hierarchy of techniques
  1. Decision procedures
  2. Rewriting (automates application of lemmas)
  3. Heuristics (guess at case-splits, instantiations)
  4. User-supplied strategies (cf. tactics in HOL)

Decision Procedures

Many important theories are decidable

- Propositional calculus

  $(a \wedge b) \vee \neg a = a \supset b$

- Equality with uninterpreted function symbols

  $x = y \wedge f(f(f(x))) = f(x) \supset f(f(f(f(f(y))))) = f(x)$

- Function, record, and tuple updates

  f **with** $[(x) := y](z) \stackrel{\text{def}}{=}$ **if** $z = x$ **then** y **else** $f(z)$

- Linear Arithmetic (over integers and rationals)

  $x \leq y \wedge x \leq 1 - y \wedge 2 \times x \geq 1 \supset 4 \times x = 2$

But we need to decide combinations of theories

$$2 \times car(x) - 3 \times cdr(x) = f(cdr(x)) \supset$$
$$f(cons(4 \times car(x) - 2 \times f(cdr(x)), y)) = f(cons(6 \times cdr(x), y))$$

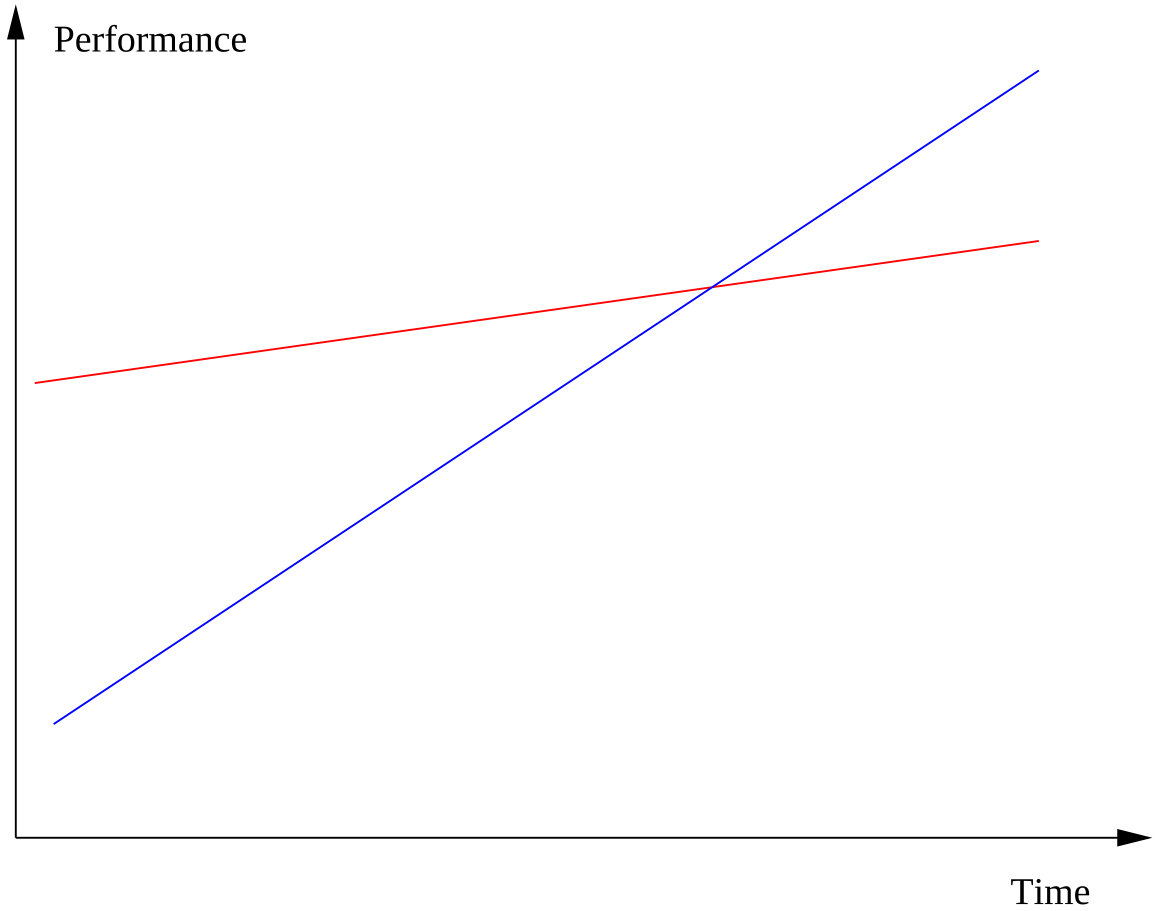
Combined Decision Procedures

- Some combinations of decidable theories are not decidable
  - E.g., quantified theory of integer arithmetic (Presburger) and equality over uninterpreted function symbols
- Need to make pragmatic compromises
  - E.g., stick to ground (unquantified) theories and leave quantification to heuristics at a higher level
- Two basic methods for combining decision procedures
  - Nelson-Oppen: fewest restrictions
  - Shostak: faster, yields a canonizer for combined theory
- Shostak's method is used in PVS
  - Over 20 years from first paper to fully correct treatment
  - Now formally verified (in PVS) by Jonathan Ford

Integrated Decision Procedures

- It's not enough to have good decision procedures available to discharge the leaves of a proof
  - The endgame: PVS can use Yices for that
- They need to be used in simplification, which involves recursive examination of subformulas: repeatedly adding, subtracting, asserting, and denying subformulas
- And integrated with rewriting, where they used in matching and (recursively) to decide conditions or top-level if-then-else's
- So the API to the decision procedures must be quite rich

Disruptive Innovation



Low-end disruption is when low-end technology overtakes the performance of high-end (Christensen)

SMT Solvers: Disruptive Innovation in Theorem Proving

- SMT stands for Satisfiability Modulo Theories
- SMT solvers extend decision procedures with the ability to handle arbitrary propositional structure
  - Traditionally, case analysis is handled heuristically in the theorem prover front end
    - Have to be careful to avoid case explosion
  - SMT solvers use the brute force of modern SAT solving
- Or, dually, they generalize SAT solving by adding the ability to handle arithmetic and other decidable theories
- Application to verification
  - Via bounded model checking and k-induction

SAT Solving

- Find satisfying assignment to a propositional logic formula
- Formula can be represented as a set of clauses
  - In CNF: conjunction of disjunctions
  - Find an assignment of truth values to variable that makes at least one literal in each clause TRUE
  - Literal: an atomic proposition A or its negation $\bar{A}$
- Example: given following 4 clauses
  - A, B
  - C, D
  - E
  - $\bar{A}, \bar{D}, \bar{E}$

  One solution is $A, C, E, \bar{D}$

  (A, D, E is not and cannot be extended to be one)
- Do this when there are 1,000,000s of variables and clauses

SAT Solvers

- SAT solving is the quintessential NP-complete problem
- But now amazingly fast in practice (most of the time)
  - Breakthroughs (starting with Chaff) since 2001
    - ⋆ Building on earlier innovations in SATO, GRASP
  - Sustained improvements, honed by competition
- Has become a commodity technology
  - MiniSAT is 700 SLOC
- Can think of it as massively effective search
  - So use it when your problem can be formulated as SAT
- Used in bounded model checking and in AI planning
  - Routine to handle 10^{300} states

SAT Plus Theories

- SAT can encode operations and relations on bounded integers
  - Using bitvector representation
  - With adders etc. represented as Boolean circuits

  And other finite data types and structures
- But cannot do not unbounded types (e.g., reals), or infinite structures (e.g., queues, lists)
- And even bounded arithmetic can be slow when large
- There are fast decision procedures for these theories
- But their basic form works only on conjunctions
- General propositional structure requires case analysis
  - Should use efficient search strategies of SAT solvers

  That's what an SMT solver does

Decidable Theories

- Many useful theories are decidable

  (at least in their unquantified forms)

  - Equality with uninterpreted function symbols

    $x = y \wedge f(f(f(x))) = f(x) \supset f(f(f(f(f(y))))) = f(x)$

  - Function, record, and tuple updates

    f **with** $[(x) := y](z) \stackrel{\text{def}}{=}$ **if** $z = x$ **then** y **else** $f(z)$

  - Linear arithmetic (over integers and rationals)

    $x \le y \wedge x \le 1 - y \wedge 2 \times x \ge 1 \supset 4 \times x = 2$

  - Special (fast) case: difference logic

    $x - y < c$

- Combinations of decidable theories are (usually) decidable

  $e.g., 2 \times car(x) - 3 \times cdr(x) = f(cdr(x)) \supset$

  $f(cons(4 \times car(x) - 2 \times f(cdr(x)), y)) = f(cons(6 \times cdr(x), y))$

  Uses equality, uninterpreted functions, linear arithmetic, lists

SMT Solving

- Individual and combined decision procedures decide conjunctions of formulas in their decided theories
- SMT allows general propositional structure
  - e.g., $(x \leq y \vee y = 5) \wedge (x < 0 \vee y \leq x) \wedge x \neq y$

    ... possibly continued for 1000s of terms
- Should exploit search strategies of modern SAT solvers
- So replace the terms by propositional variables
  - i.e., $(A \vee B) \wedge (C \vee D) \wedge E$
- Get a solution from a SAT solver (if none, we are done)
  - e.g., A, D, E
- Restore the interpretation of variables and send the conjunction to the core decision procedure
  - i.e., $x \leq y \wedge y \leq x \wedge x \neq y$

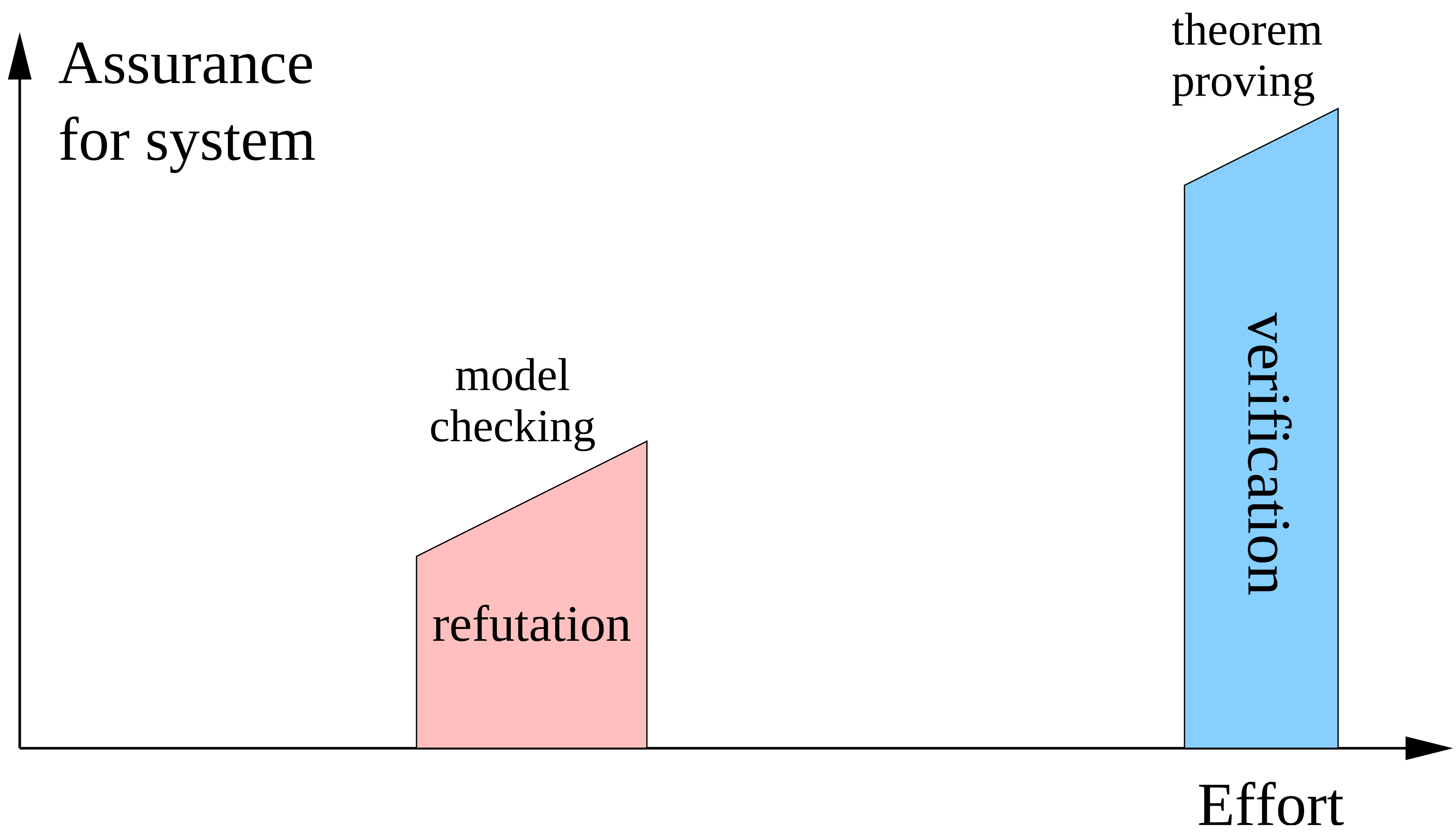
SMT Solving by "Lemmas On Demand"

- If satisfiable, we are done
- If not, ask SAT solver for a new assignment
- But isn't it expensive to keep doing this?
- Yes, so first, do a little bit of work to find fragments that explain the unsatisfiability, and send these back to the SAT solver as additional constraints (i.e., lemmas)
  - $A \wedge D \supset \bar{E}$ (equivalently, $\bar{A} \vee \bar{D} \vee \bar{E}$)
- Iterate to termination
  - e.g., $A, C, E, \bar{D}$
  - i.e., $x \leq y, x < 0, x \neq y, y \not\leq x$ (simplifies to $x < y, x < 0$)
  - A satisfying assignment is $x = -3, y = 1$
- This is called "lemmas on demand" (de Moura, Ruess, Sorea) or "DPLL(T)"; it yields effective SMT solvers

Pros and Cons of Model Checking

- "*Model checking saved the reputation of formal methods*"
- But have to be explicit where we may prefer not to be
  - E.g., have to specify the ALU (Arithmetic Logic Unit) when we're really only interested in the pipeline logic
  - But infinite BMC allows use of uninterpreted functions
- Usually have to downscale the model—can be a lot of work
- Often good at finding bugs, but what if no bugs detected?
  - Have we achieved verification, or just got too crude a model or property?
- Sometimes it's possible to prove that a small model is a property-preserving abstraction of a large, accurate one
- Then not detecting a bug is equivalent to verification

Model Checking: An Island



Combining Model Checking and Theorem Proving

- Model checking a downscaled instance is a useful prelude to theorem proving the general case
- But a more interesting combination is to use model checking as part of a proof for the general case
- One approach is to create a finite state property-preserving abstraction of the original protocol
  - Theorem proving shows abstraction preserves the property
  - Model checking shows abstraction satisfies the property

    Instead of proving `indinv(safe)`, we invoke a model checker to show `abs_system |- G(safe)` [LTL]

  Can actually do all of this within PVS, because it includes a symbolic model checker (for CTL)

  - Built on a decision procedure for finite μ-calculus
  - We use it to prove `init(s) => AG(safe)(s)` [CTL]

Conditions for Property-Preserving Abstraction

- Want an abstracted state type `abstract_state`
  - And corresponding transition relation `a_trans`
  - And initiality predicate `a_init`
  - Together with abstracted safety property `a_safe`
- And an abstraction function `abst` from `state` to `abstract_state`, such that following properties hold

```
init_simulation: THEOREM
  init(s) IMPLIES a_init(abst(s))
trans_simulation: THEOREM
  transitions(pre,post) IMPLIES a_trans(abst(pre),abst(post))
safety_preserved: THEOREM
  a_safe(abst(s)) IMPLIES safe(s)
abs_invariant_ctl: THEOREM      % a_safe is invariant
  a_init(as) IMPLIES AG(a_trans, a_safe)(as)
```

Abstracted Model

- It doesn't matter to the protocol what the actual values of the tickets are
- All that matters is whether or not each of them is zero, and whether one is less than the other
- We can use Booleans to represent these relations
  - This is called predicate abstraction
- So introduce the abstracted (finite) state type

```
abstract_state: TYPE =
  [# pc1, pc2: phase,
     t1_is_0, t2_is_0, t1_lt_t2: bool #]

  as, a_pre, a_post: VAR abstract_state
```

Abstraction Function
And Abstracted Properties in PVS

```
abst(s): abstract_state =
  (# pc1 := s`pc1, pc2 := s`pc2,
     t1_is_0 := s`t1 = 0, t2_is_0 := s`t2 = 0,
     t1_lt_t2 := s`t1 < s`t2 #)

a_init(as): bool =
   as`pc1 = idle AND as`pc2 = idle
     AND as`t1_is_0 AND as`t2_is_0

a_safe(as): bool =
   NOT (as`pc1 = critical AND as`pc2 = critical)
```

More of the Abstracted Specification

```
a_P1_transition(a_pre, a_post): bool =
 IF a_pre`pc1 = idle
   THEN a_post = a_pre WITH [(t1_is_0) := false,
                             (t1_lt_t2) := false,
                             (pc1) := trying]
 ELSIF a_pre`pc1 = trying
         AND (a_pre`t2_is_0 OR a_pre`t1_lt_t2)
   THEN a_post = a_pre WITH [(pc1) := critical]
 ELSIF a_pre`pc1 = critical
   THEN a_post = a_pre WITH [(t1_is_0) := true,
                             (t1_lt_t2) := NOT a_pre`t2_is_0,
                             (pc1) := idle]
 ELSE a_post = a_pre
ENDIF
```

The Rest of the Abstracted Specification

```
a_P2_transition(a_pre, a_post): bool =
 IF a_pre`pc2 = idle
   THEN a_post = a_pre WITH [(t2_is_0) := false,
                             (t1_lt_t2) := true,
                             (pc2) := trying]
 ELSIF a_pre`pc2 = trying
        AND (a_pre`t1_is_0 OR NOT a_pre`t1_lt_t2)
   THEN a_post = a_pre WITH [(pc2) := critical]
 ELSIF a_pre`pc2 = critical
   THEN a_post = a_pre WITH [(t2_is_0) := true,
                             (t1_lt_t2) := false,
                             (pc2) := idle]
 ELSE a_post = a_pre
ENDIF
```

Proofs to Justify The Abstraction

- The conditions `init_simulation` and `safety_preserved` are proved by `(GRIND)`
- And `abs_invariant_ctl` is proved by `(MODEL-CHECK)`
  - Or could use `sal-smc` on the SAL equivalent
- But `trans_simulation` has 2 unproved cases—here's the first

```
trans_simulation.2.6.1 :
[-1]  post!1 = pre!1
{-2}  idle?(pre!1`pc1)
  |-------
{1}   critical?(pre!1`pc2)
[2]   pre!1`t1 = 0
[3]   pre!1`t1 > pre!1`t2
{4}   idle?(pre!1`pc2)
{5}   pre!1`t1 < pre!1`t2
```

The Problem Justifying The Abstraction

- The problem here is that when the two tickets are equal but nonzero, the concrete protocol drops through to the `ELSE` case and requires the pre and post states to be the same
- But in the abstracted protocol, this situation can satisfy the condition for Process 2 to enter its critical section
  - Because `NOT a_pre‘t1_lt_t2` abstracts `pre‘t1 >= pre‘t2` rather than `pre‘t1 > pre‘t2`
- But this situation can never arise, because each ticket is always incremented to be strictly greater than the other
- We can prove this as an invariant

```
not_eq(s): bool = s‘t1 = s‘t2 => s‘t1 = 0

extra: LEMMA indinv(not_eq)
```

- This is proved with `(GRIND)`

A Justification of the Abstraction

- A stronger version of the simulation property allows us to use a known invariant to establish it

```
strong_trans_simulation: THEOREM
   indinv(not_eq)
    AND not_eq(pre) AND not_eq(post)
    AND transitions(pre, post)
      IMPLIES a_trans(abst(pre), abst(post))
```

- This is proved by

```
 (SKOSIMP)
 (EXPAND "transitions")
 (GROUND)
 (("1" (EXPAND "P1_transition")
       (APPLY (THEN (GROUND) (GRIND))))
  ("2" (EXPAND "P2_transition")
       (APPLY (THEN (GROUND) (GRIND)))))
```

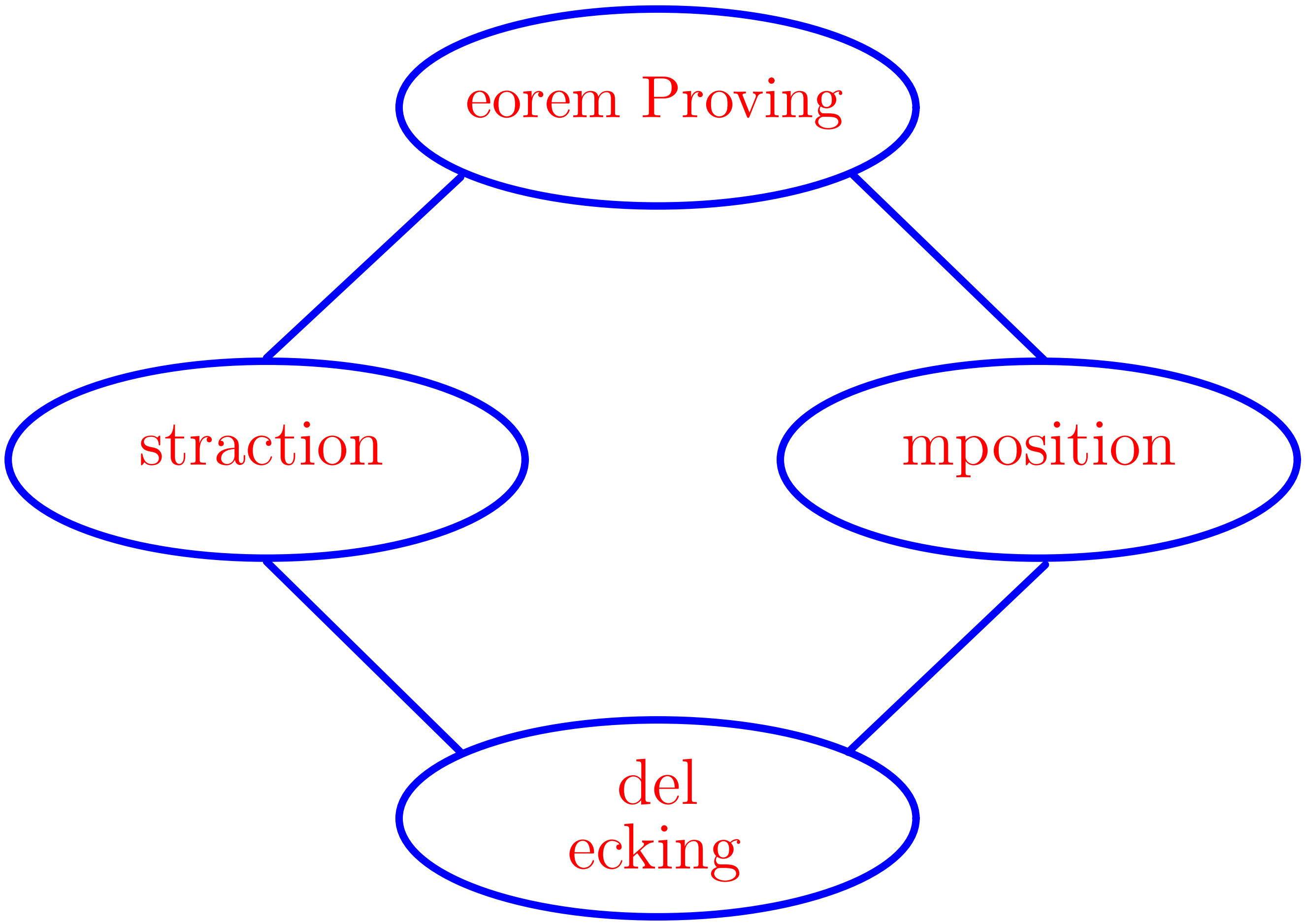
Pros and Cons of Manually-Constructed Abstractions

- Justifying the abstraction is usually almost as hard as proving the property directly
- And generally requires auxiliary invariants
- Bounded retransmission protocol required 45 of the original 57 invariants to justify an abstraction
- But there's the germ of an idea here

Abstraction Is The Bridge

Between Deductive and Algorithmic Methods

And Between Refutation and Verification



Failure-Tolerant Theorem Proving

- Model checking is based on search
- Safe to do because the search space is bounded, and efficient because we know its structure
- Verification systems (theorem provers aimed at verification) tend to avoid search at the top level
    - Too big a space to search, too little known about it
    - When they do search, they have to rely on heuristics
    - Which often fail
- Classical verification poses correctness as one "big theorem"
    - So failure to prove it (when true) is catastrophic
- Instead, let's try "failure-tolerant" theorem proving
    - Prove lots of small theorems instead of one big one
    - In a context where some failures can be tolerated

Contexts for Failure-Tolerant Theorem Proving

- Extended static checking (see later)
- Property preserving abstractions
  - Instead of justifying an abstraction,
  - Use deduction to calculate it
- Given a transition relation G on S and property P, a property-preserving abstraction yields a transition relation $\hat{G}$ on $\hat{S}$ and property $\hat{P}$ such that

$$\hat{G} \models \hat{P} \Rightarrow G \models P$$

  Where $\hat{G}$ and $\hat{P}$ that are simple to analyze (e.g., finite state)
- A good abstraction typically (for safety properties) introduces nondeterminism while preserving the property
- Note that abstraction is not the inverse of refinement

Calculating an Abstraction

- We need to figure out if we need a transition between any pair of abstract states
- Given abstraction function $\phi : [S \rightarrow \hat{S}]$ we have

$$\hat{G}(\hat{s}_1, \hat{s}_2) \Leftrightarrow \exists s_1, s_2 : \hat{s}_1 = \phi(s_1) \wedge \hat{s}_2 = \phi(s_2) \wedge G(s_1, s_2)$$

- We'll use highly automated theorem proving on these formulas: include transition iff the formula is proved
  - There's a chance we may fail to prove true formulas
  - This will produce unsound abstractions
- So turn the problem around and calculate when we don't need a transition: omit transition iff the formula is proved

$$\neg\hat{G}(\hat{s}_1, \hat{s}_2) \Leftrightarrow \vdash \forall s_1, s_2 : \hat{s}_1 \neq \phi(s_1) \vee \hat{s}_2 \neq \phi(s_2) \vee \neg G(s_1, s_2)$$

- Now theorem-proving failure affects accuracy, not soundness

Automated Abstraction

- The method described is automated in InVeSt
  - An adjunct to PVS developed in conjunction with Verimag
- A different method (due to Saïdi and Shankar) is implemented in PVS
  - Exponentially more efficient
- The abstraction is specified in the proof command by giving the concrete function or predicate that defines the value of each abstract state variable

Automated Abstraction in PVS

- `auto_abstract: THEOREM`

  ```
  init(s) IMPLIES AG(transitions, safe)(s)
  ```

- This is proved by

  ```
  (abstract-and-mc "state" "abstract_state"
   (("t1_is_0" "lambda (s): s`t1=0")
    ("t2_is_0" "lambda (s): s`t2=0")
    ("t1_lt_t2" "lambda (s): s`t1 < s`t2")))
  ```

- Now let's see this work in practice

Other Kinds of Abstraction

- We've seen predicate abstraction [Graf and Saïdi]
- I'll briefly sketch data abstraction and hybrid abstraction

Data Abstraction [Cousot & Cousot]

- Replace concrete variable x over datatype C by an abstract variable x' over datatype A through a mapping $h : [C \rightarrow A]$
- Examples: Parity, $mod\ N$, zero-nonzero, intervals, cardinalities, {0, 1, many}, {empty, nonempty}
- Syntactically replace functions f on C by abstracted functions $\hat{f}$ on A
- Given $f : [C \rightarrow C]$, construct $\hat{f} : [A \rightarrow set[A]]$:
(observe how data abstraction introduces nondeterminism)

$$b \in \hat{f}(a) \Leftrightarrow \exists x : a = h(x) \wedge b = h(f(x))$$

$$b \notin \hat{f}(a) \Leftrightarrow \ \vdash \forall x : a = h(x) \Rightarrow b \neq h(f(x))$$

- Theorem-proving failure affects accuracy, not soundness

Data Abstraction Example

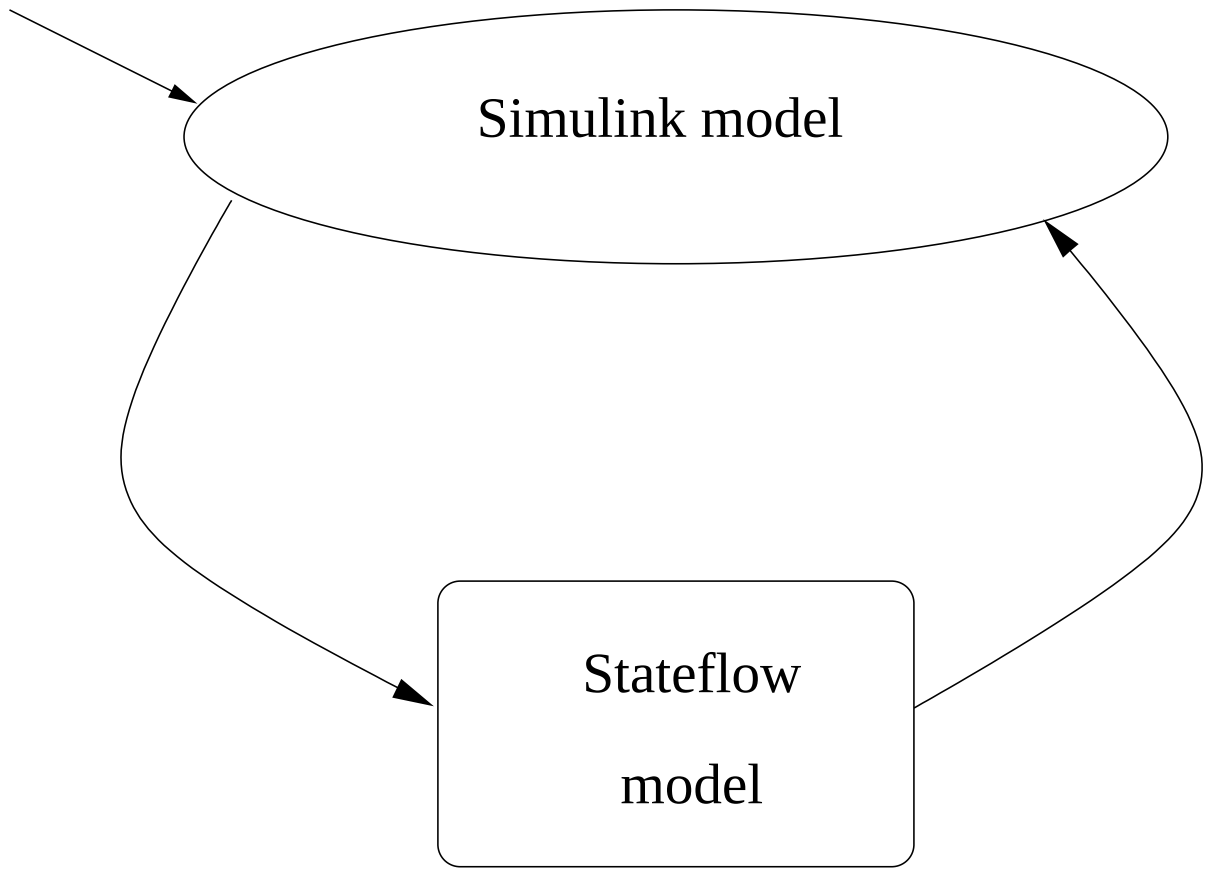
Replace natural numbers by {0, 1, many}

Calculate behavior of subtraction on {0, 1, many}

$-$	0	1	many
0	0	$-$	$-$
1	1	0	$-$
many	many	$\{1, \text{many}\}$	$\{0, 1, \text{many}\}$

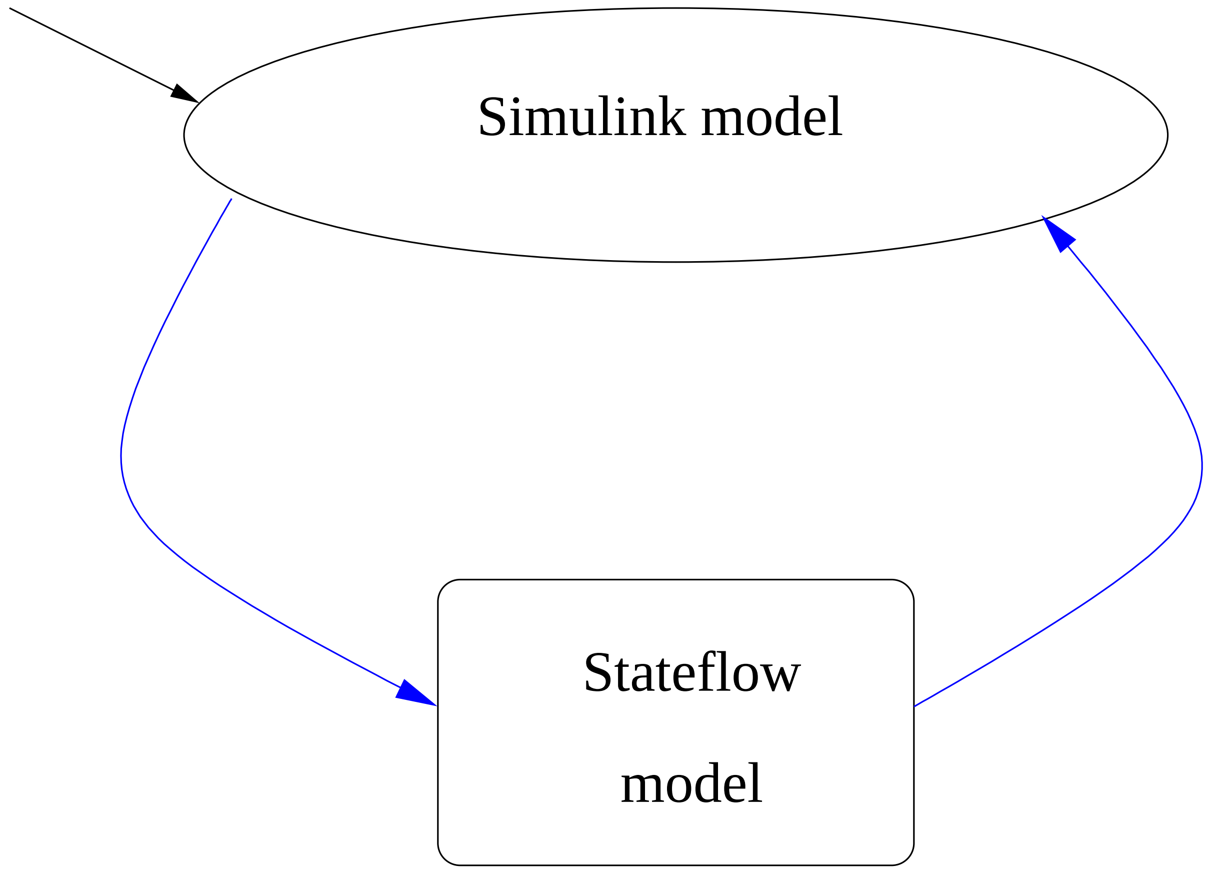
$$0 \notin (\text{many} - 1) \ \textit{iff} \ \forall x \in \{2, 3, 4, \ldots\} : x - 1 \neq 0$$

Data Abstraction for Matlab (Hybrid Systems)



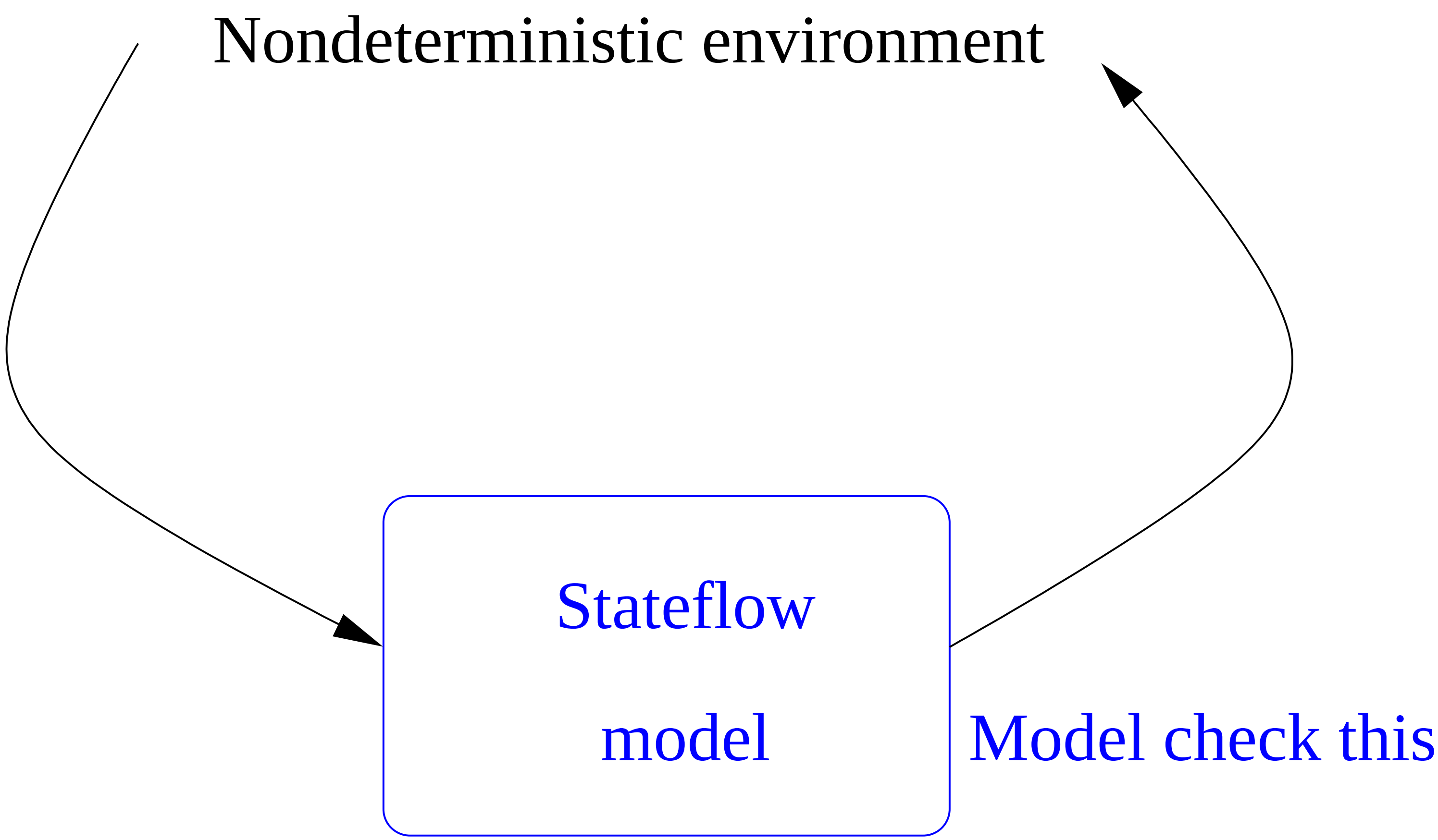
Mixed continuous/discrete (i.e., hybrid) system

Simulate One Trajectory at a Time



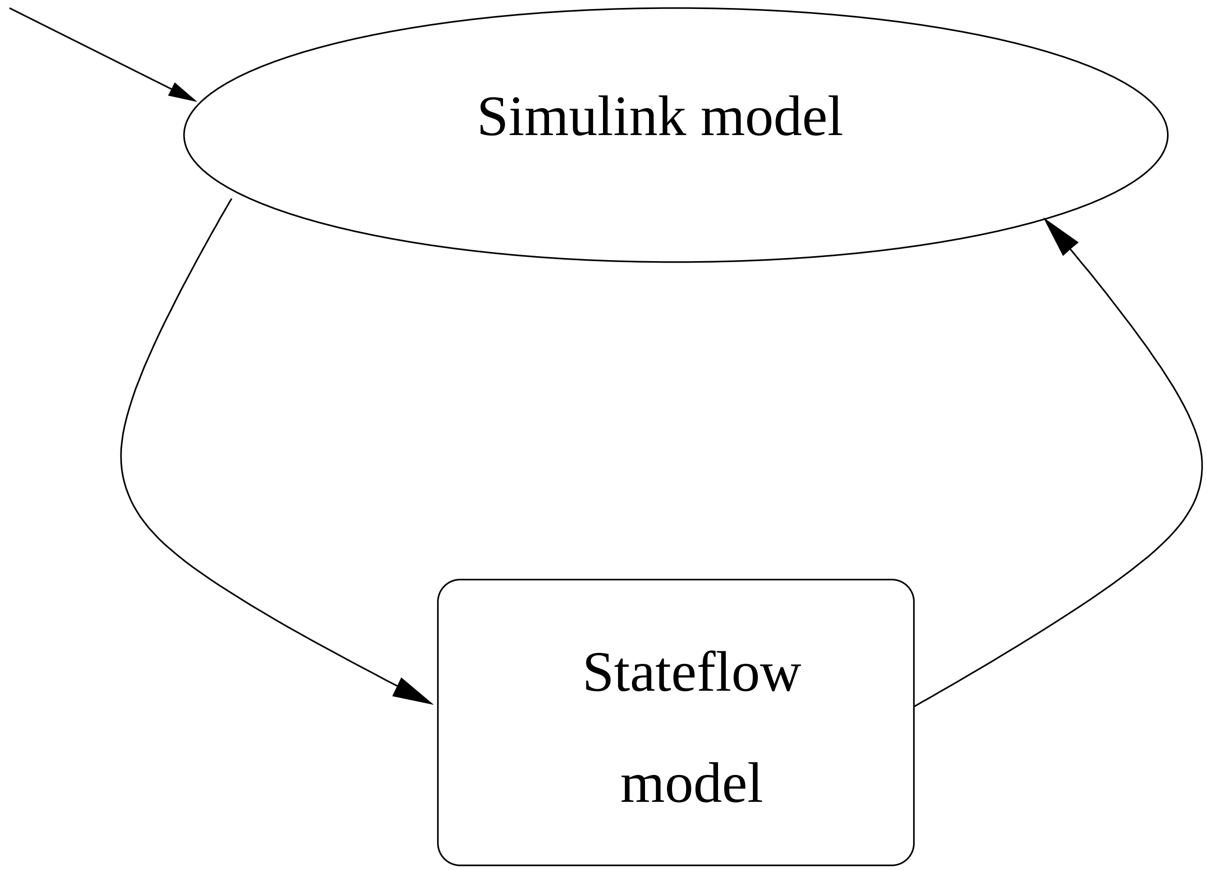
Just like testing: when have you done enough?

Model Check With Nondeterministic Environment



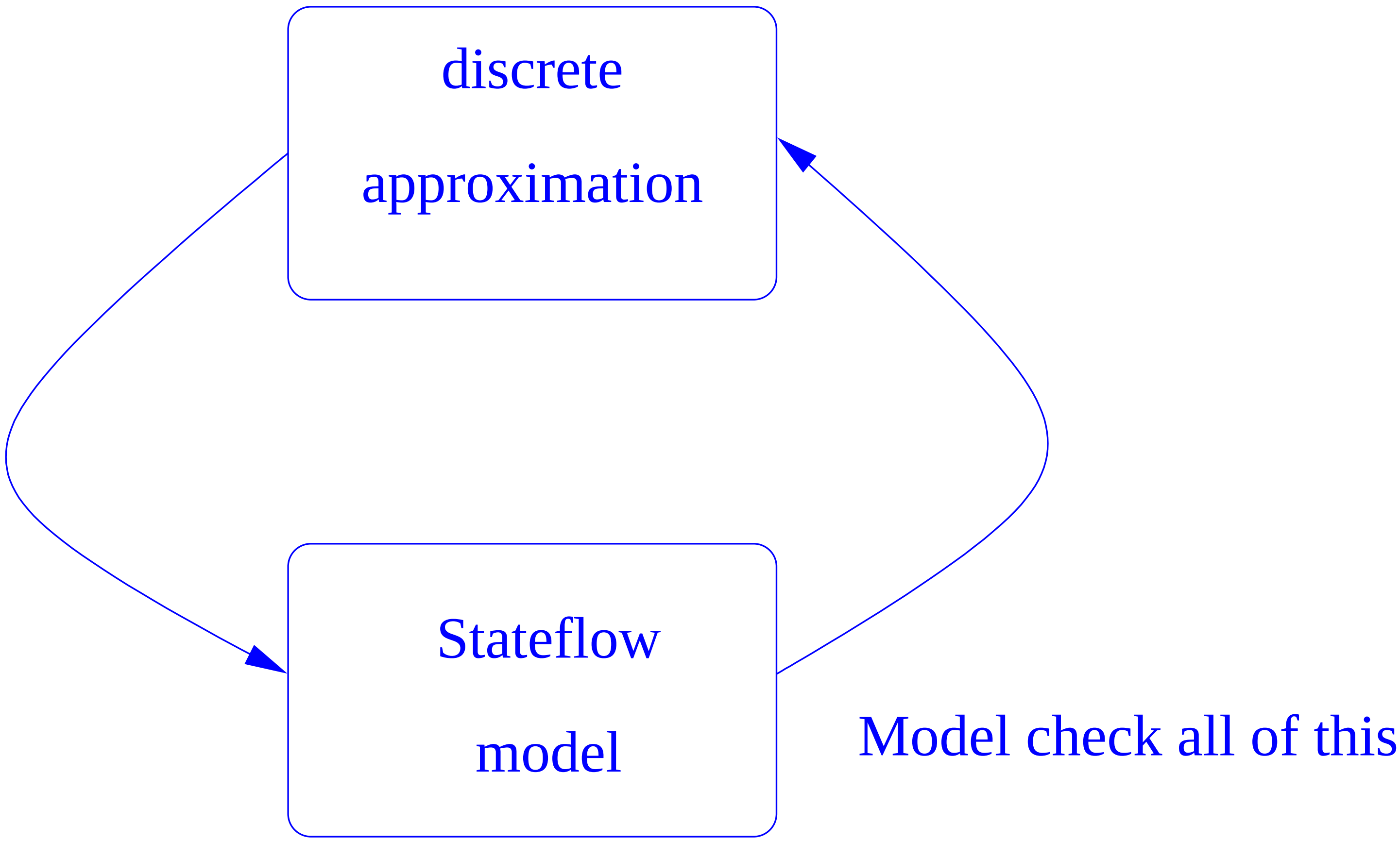
Too crude to establish useful properties

Analyze By The Methods Of Hybrid Systems



OK, but restricted

Model Check With Sound Discretization Of The Continuous Environment



Just right

Data Abstraction for Hybrid Systems

- Method developed by Ashish Tiwari
- The continuous environment is given by some collection of (polynomial) differential equations on $\mathbf{R}^n$
- Divide these into regions where the first j derivatives are sign-invariant (m polynomials, $(m \times j)^3$ regions)
    - I.e., data abstraction from $\mathbf{R}$ to $\{-, 0, +\}$
    - For each mode $l \in \mathbf{Q}$: if q_{pi}, q_{pj} abstract p_i, p_j and $\dot{p_i} = p_j$ in mode l, then apply rules of the form:
        - ⋆ if $q_{pi} = +$ & $q_{pj} = +$, then q'_{pi} is $+$
        - ⋆ if $q_{pi} = +$ & $q_{pj} = 0$, then q'_{pi} is $+$
        - ⋆ if $q_{pi} = +$ & $q_{pj} = -$, then q'_{pi} is either $+$ or 0
        - ⋆ ...

Data Abstraction for Hybrid Systems

- Larger choices of j give successively finer abstractions
- Usually enough to take $j = 1$ or 2
- Method is complete for some (e.g., nilpotent) systems
- Parameterized also by selection of polynomials to abstract on
  - The eigenvectors are a good start
  - Method is then complete for linear systems
- Construction is automated using decision procedures for real closed fields (e.g., Cylindric Algebraic Decomposition—CAD)
- Also provides a general underpinning to qualitative reasoning as used in AI

Example: Thermostat

Consider a simple thermostat controller with:

- Discrete modes: Two modes, $q = on$ and $q = \mathit{off}$
- Continuous variable: The temperature x
- Initial State: $q = \mathit{off}$ and $x = 75$
- Discrete Transitions:

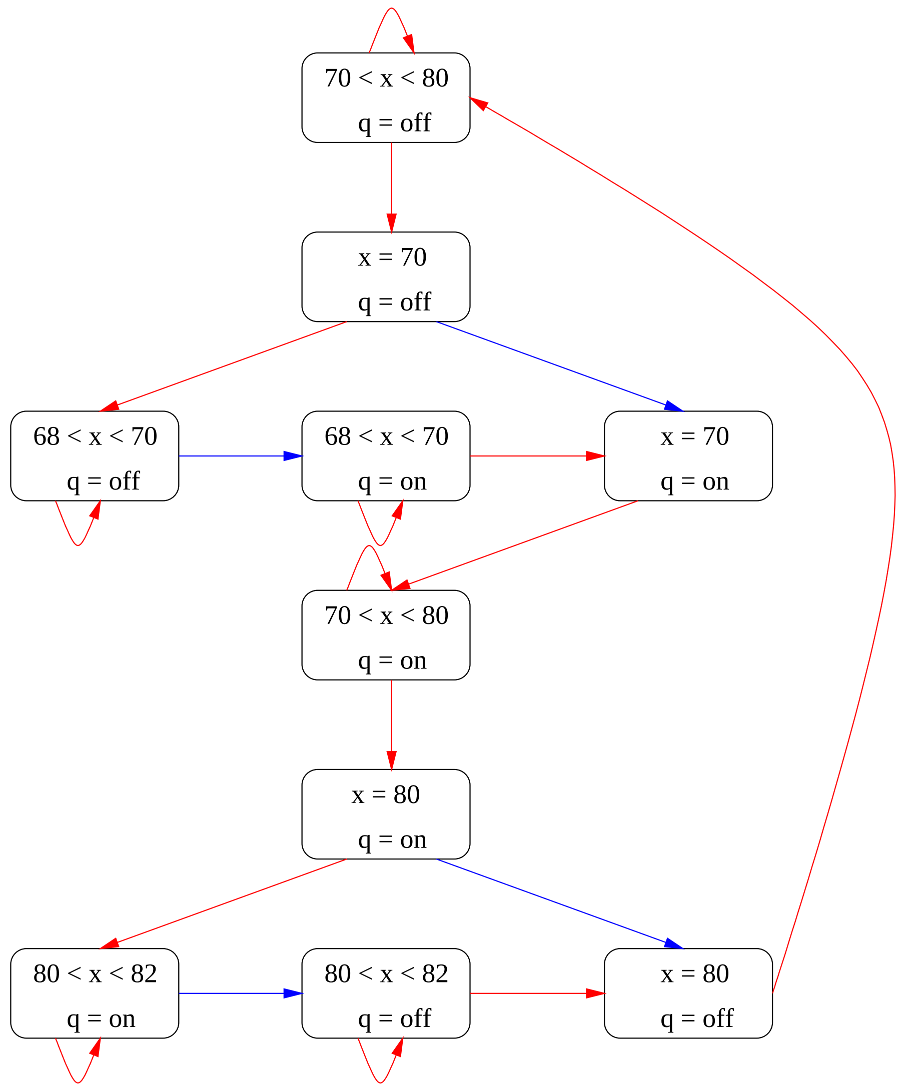
$$
\begin{array}{rcl}
q = \mathit{off} \ \mathit{and} \ x \leq 70 & \longrightarrow & q' = on \\
q = on \ \mathit{and} \ x \geq 80 & \longrightarrow & q' = \mathit{off}
\end{array}
$$

- Continuous Flow:

$$
\begin{array}{rcl}
q = \mathit{off} \ \mathit{and} \ x > 68 & \longrightarrow & \dot{x} = -Kx \\
q = on \ \mathit{and} \ x < 82 & \longrightarrow & \dot{x} = K(h - x)
\end{array}
$$

We want to prove $68 \leq x \leq 82$

Abstract Thermostat System



Pros and Cons of Automated Abstraction

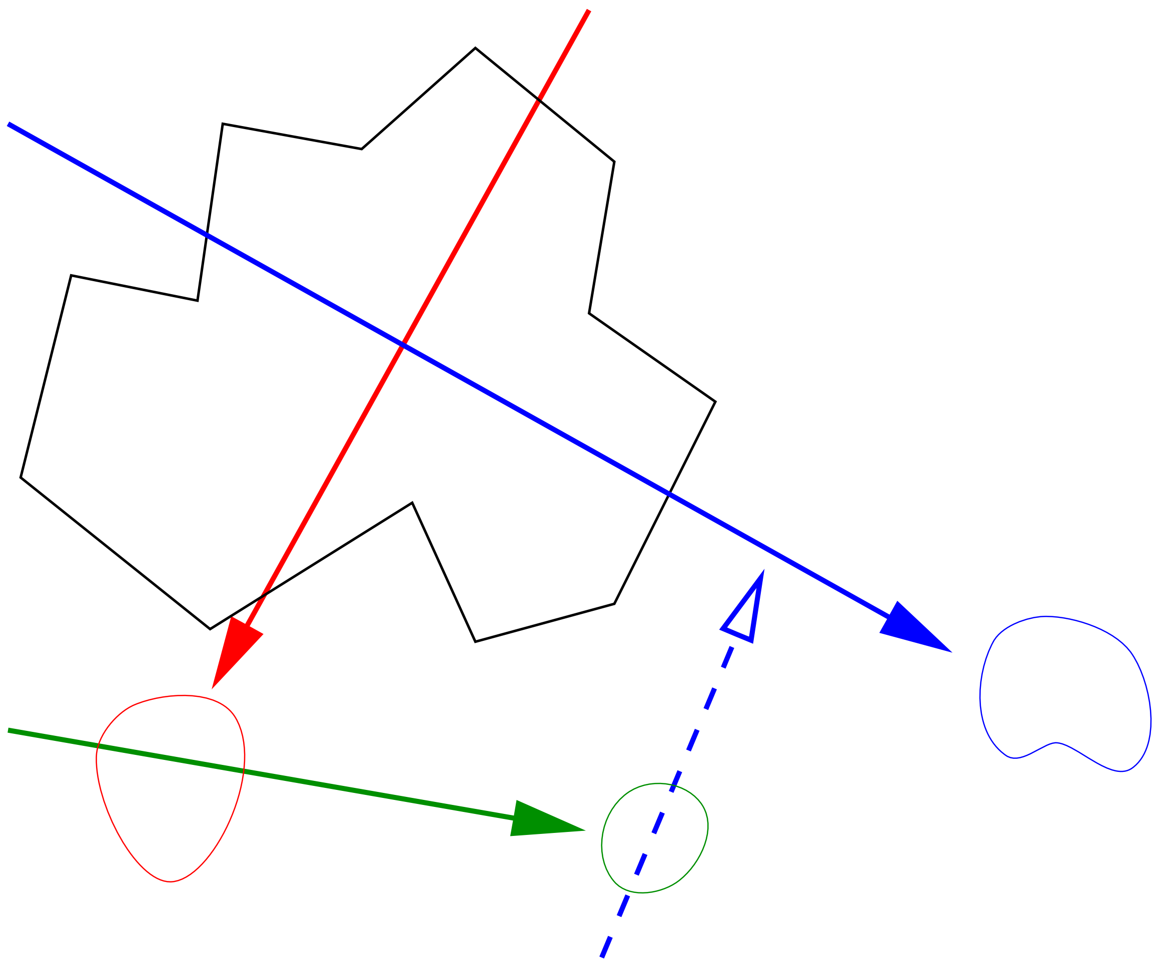
- Good match between local theorem proving, and global model checking
- Quality of the abstraction depends on information provided by the user (predicates, polynomials etc.)
  - It's easier to guess useful predicates than invariants
  - Can guess additional ones if inadequate
  - Or let counterexamples suggest refinements (CEGAR)
    - ⋆ A general approach can be discerned here: find quick solutions and fix them up, rather than deliberate in hope of finding good solutions

  And the deductive power applied
  - Which may increase if provided with known invariants

Truly Integrated, Iterated Analysis!

- Recast the goal as one of calculating and accumulating properties about a design (symbolic analysis)
- Rather than just verifying or refuting a specific property
- Properties convey information and insight, and provide leverage to construct new abstractions
    - And hence more properties
- Requires restructuring of verification tools
    - So that many work together
    - And so that they return symbolic values and properties rather than just yes/no results of verifications
- This is what SAL is about: Symbolic Analysis Laboratory
    - Next generation will have a tool bus

Integrated, Iterated Analysis



Refutation and Verification

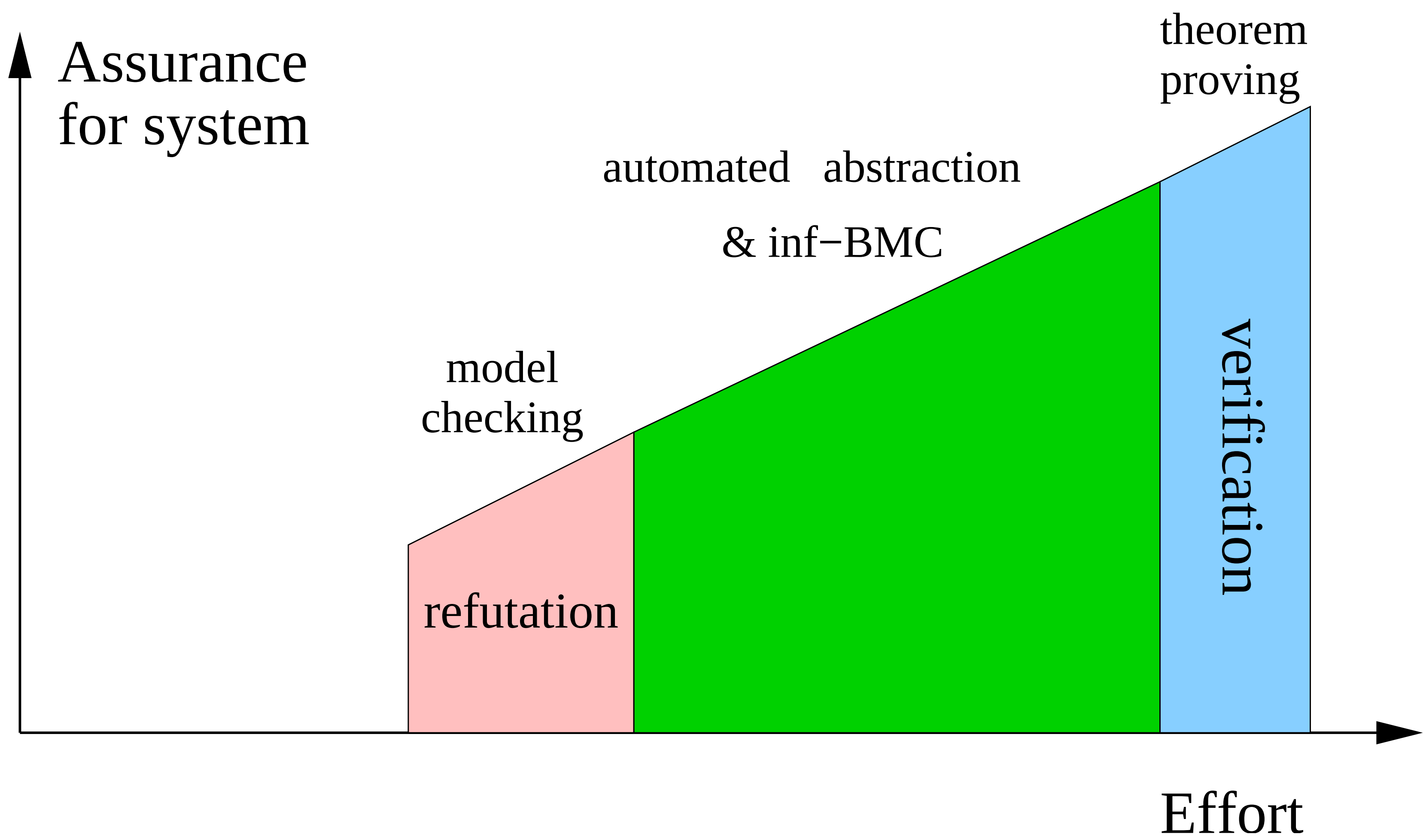
- By allowing unsound abstractions

$$\hat{G} \models \hat{P} \not\Rightarrow G \models P$$

  We can do refutation as well as verification

- Then, by selecting abstractions (sound/unsound) and properties (little/big) we can fill in the space between refutation and verification
- Refutation lowers the barrier to entry
- Provides economic incentive: discovery of high value bugs
  - Can estimate the cost of each bug found
  - And can directly compare with other technologies
- Yet allows smooth transition to verification

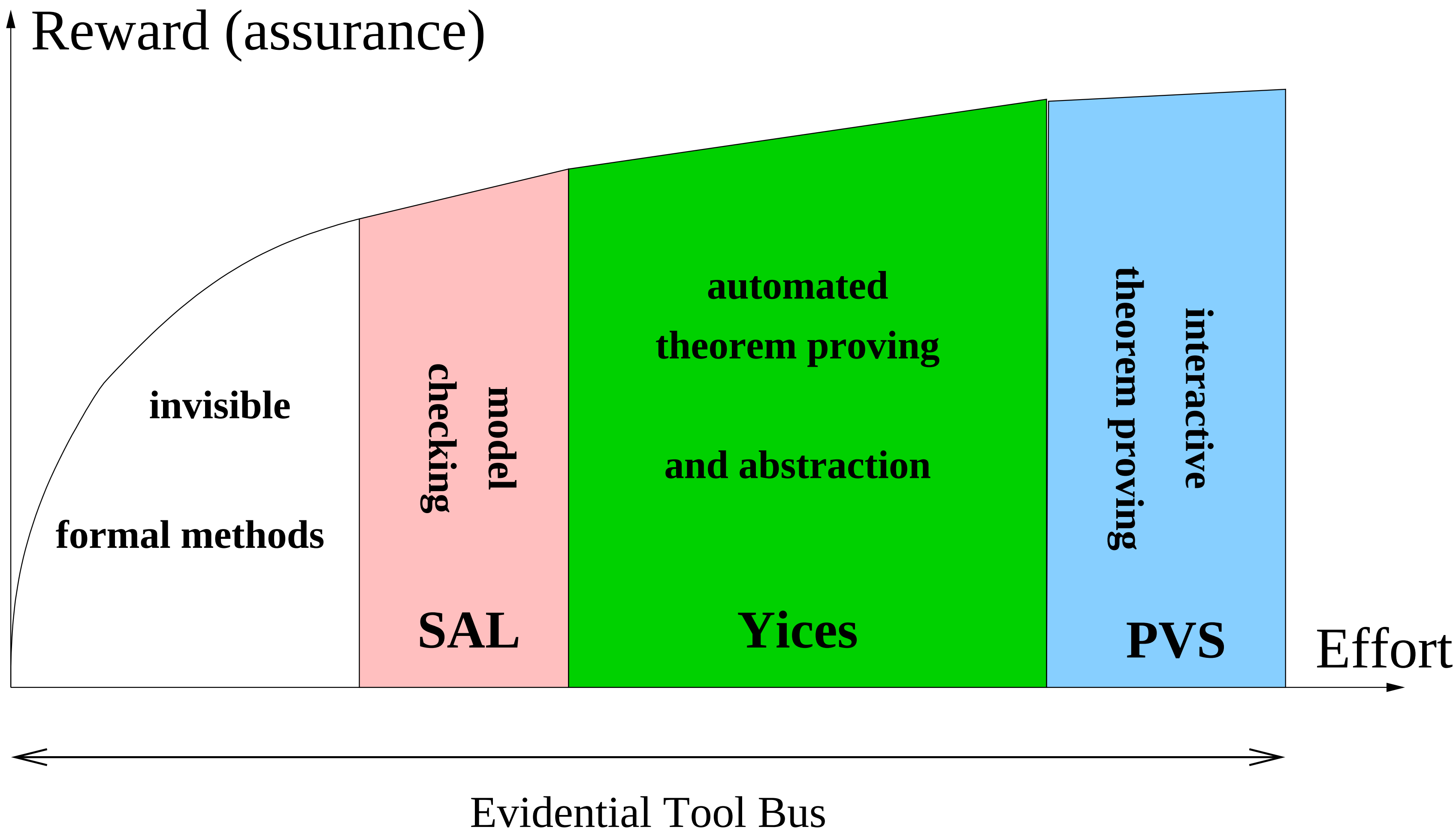
From Refutation To Verification



Filling the Remaining Gap

- Model checking for refutation and (via automated abstraction and inf-BMC) for verification imposes a much smaller barrier to adoption than old-style formal verification
- But the barrier is still there
- What about really low cost/low threat kinds of formal analysis?
- Make the formal methods disappear inside traditional tools and methods
    - We call these invisible formal methods
    - And it's where a lot of the action and opportunity is

The Formal Methods Wedge



Examples of Invisible Formal Methods

Stronger Checking in Traditional Tools

- Various forms of extended static checking
  - Failed proof generates a possibly spurious warning
- Static analysis: typestate, shape analysis, abstract interpretation etc.
- PVS-like type system (predicate subtypes) for any language
  - Traditional type systems have to be trivially decidable
  - But can gain enormous error detection by adding a component that requires theorem proving (lots of small theorems, failure generates a spurious warning)
- The verifying compiler (aka. verified systems roadmap)

Examples of Invisible Formal Methods

Better Tools for Traditional Activities

- Statechart/Stateflow property checkers
  (cf. Reactis, Honeywell, SRI, Mathworks)
  - Show me a path that activates this state
  - Can this state and that be active simultaneously?
- Checker synthesizers (cf. IBM FOCS)
- Completeness/Consistency checkers for tabular specifications
  (cf. Ontario Hydro, RSML, SCR)
- Test case generators (cf. Verimag/IRISA TGV and STG, SAL-ATG)

There's an entire industry in this space, with many companies make a living from modest technology (but very good understanding of their markets)

Static Program Analysis

The Bug That Stopped The Zunes

Real time clock sets `days` to number of days since 1 Jan 1980

```
year = ORIGINYEAR; /* = 1980 */

while (days > 365) {
    if (IsLeapYear(year)) {
        if (days > 366) {
            days -= 366;
            year += 1;
        } else... loops forever on last day of a leap year
    } else {
        days -= 365;
        year += 1;
    }
}
```

Coverage-based testing will find this

A Hasty Fix

```
while (days > 365) {
    if (IsLeapYear(year)) {
        if (days > 365) {
            days -= 366;
            year += 1;
        }
    } else {
        days -= 365;
        year += 1;
    }
}
```

- Fixes the loop but now `days` can end up as zero
- Coverage-based testing might not find this
- Boundary condition testing would
- But I think the point is clear...

The Problem With Testing

- Is that it only samples the set of possible behaviors
- And unlike physical systems (where many engineers gained their experience), software systems are discontinuous
- There is no sound basis for extrapolating from tested to untested cases
- So we need to consider all possible cases. . . how is this possible?
- It's possible with symbolic methods
- Cf. $x^2 - y^2 = (x - y)(x + y)$ vs. `5*5-3*3 = (5-3)*(5+3)`
- Static Analysis is about totally automated ways to do this

The Zune Example Again

```
[days > 0]
while (days > 365) {       [days > 365]
    if (*)) {
        if (days > 365) {  [days > 365]
            days -= 366;   [days >= 0]
            year += 1;
        }
    } else {               [days > 365]
        days -= 365;       [days > 0]
        year += 1;
    }
}
[days >= 0 and days <= 365]
```

Automated Analysis of the Zune Example

- Ashish Tiwari has an analyzer for C programs that does abstract interpretation over the combination of linear arithmetic and uninterpreted functions
- Primarily used to discover invariants
- But we can illustrate it on this example

```
abs-li zune.c

--------------------FINAL Invariants--------------------

days >= 1.000000  days <= 365.000000

loop >= 0.000000  loop <= 1.000000
```

  Monitoring variable loop can equal 1 means program can loop
- Now the hastily fixed version

```
abs-li zune2.c

--------------------FINAL Invariants--------------------

days >= 0.000000  days <= 365.000000

loop = 0.000000
```

  Now it does not loop, but days can be 0

Approximations

- We were lucky that we could do the previous example with full symbolic arithmetic
- Usually, the formulas get bigger and bigger as we accumulate information from loop iterations (we'll see an example later)
- So it's common to approximate or abstract information to try and keep the formulas manageable
- Here, instead of the natural numbers 0, 1, 2, ..., we could use
    - `zero, small, big`
    - Where `big` abstracts everything bigger than 365, `small` is everything from 1 to 365, and `zero` is 0
    - Arithmetic becomes nondeterministic
        - ⋆ e.g., `small+small` = `small | big`

The Zune Example Abstracted

```
[days = small | big]
while (days = big) {         [days = big]
    if (*)) {
        if (days = big ) {  [days = big ]
            days -= big;    [days = big | small | zero]
            year += 1;
        }
    } else {                 [days = big]
        days -= small;       [days = big | small]
        year += 1;
    }
}
[days = small | zero]
```

The Zune Example Abstracted Again

Suppose we abstracted to {negative, zero, positive}

```
[days = positive]
while (days = positive) {          [days = positive]
    if (*)) {
        if (days = positive ) {    [days = positive ]
            days -= positive;      [days = negative | zero | positive]
            year += 1;
        }
    } else {                       [days = positive]
        days -= positive;          [days = negative | zero | positive]
        year += 1;
}   }
[days = negative | zero]
```

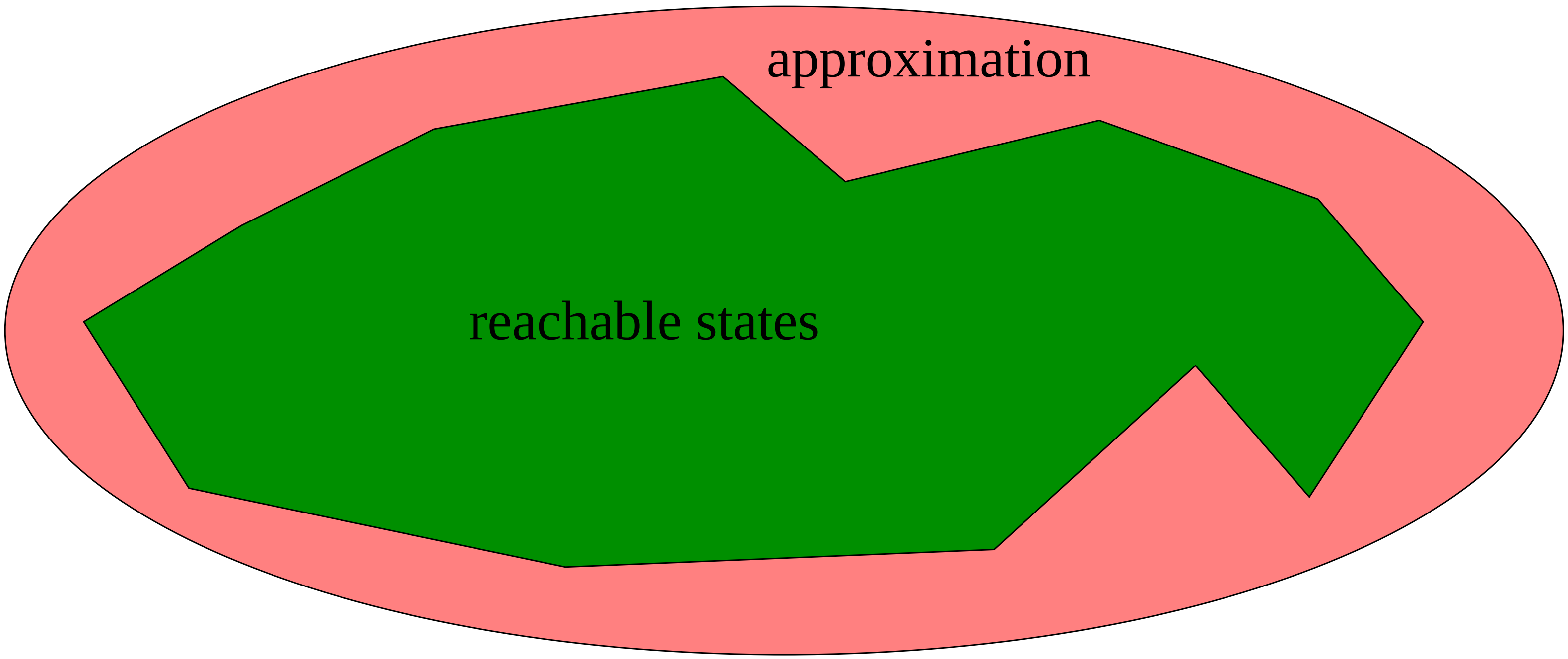
We've lost too much information: have a false alarm that `days` can go negative (pointer analysis is sometimes this crude)

We Have To Approximate, But There's A Price

- It's no accident that we sometimes lose precision
- Rice's Theorem says there are inherent limits on what can be accomplished by automated analysis of programs
  - Sound (miss no errors)
  - Complete (no false alarms)
  - Automatic
  - Allow arbitrary (unbounded) memory structures
  - Final results

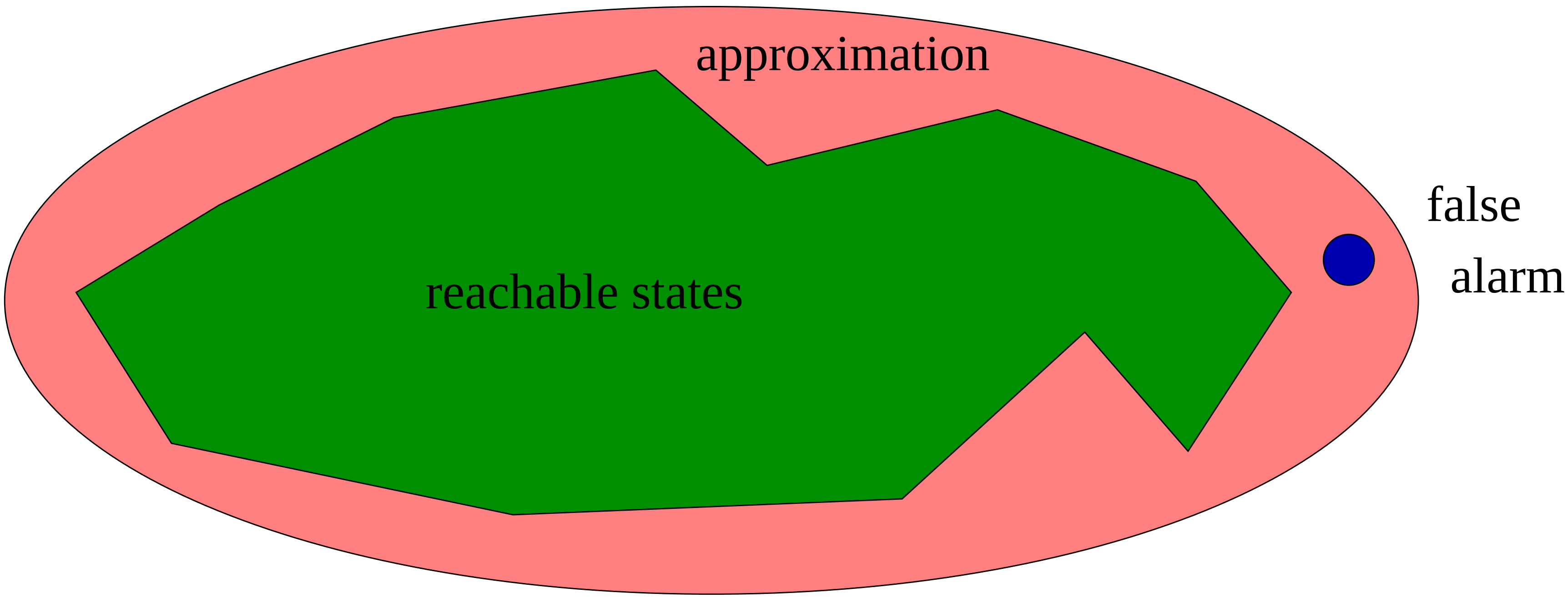
  Choose at most 4 of the 5

Approximations



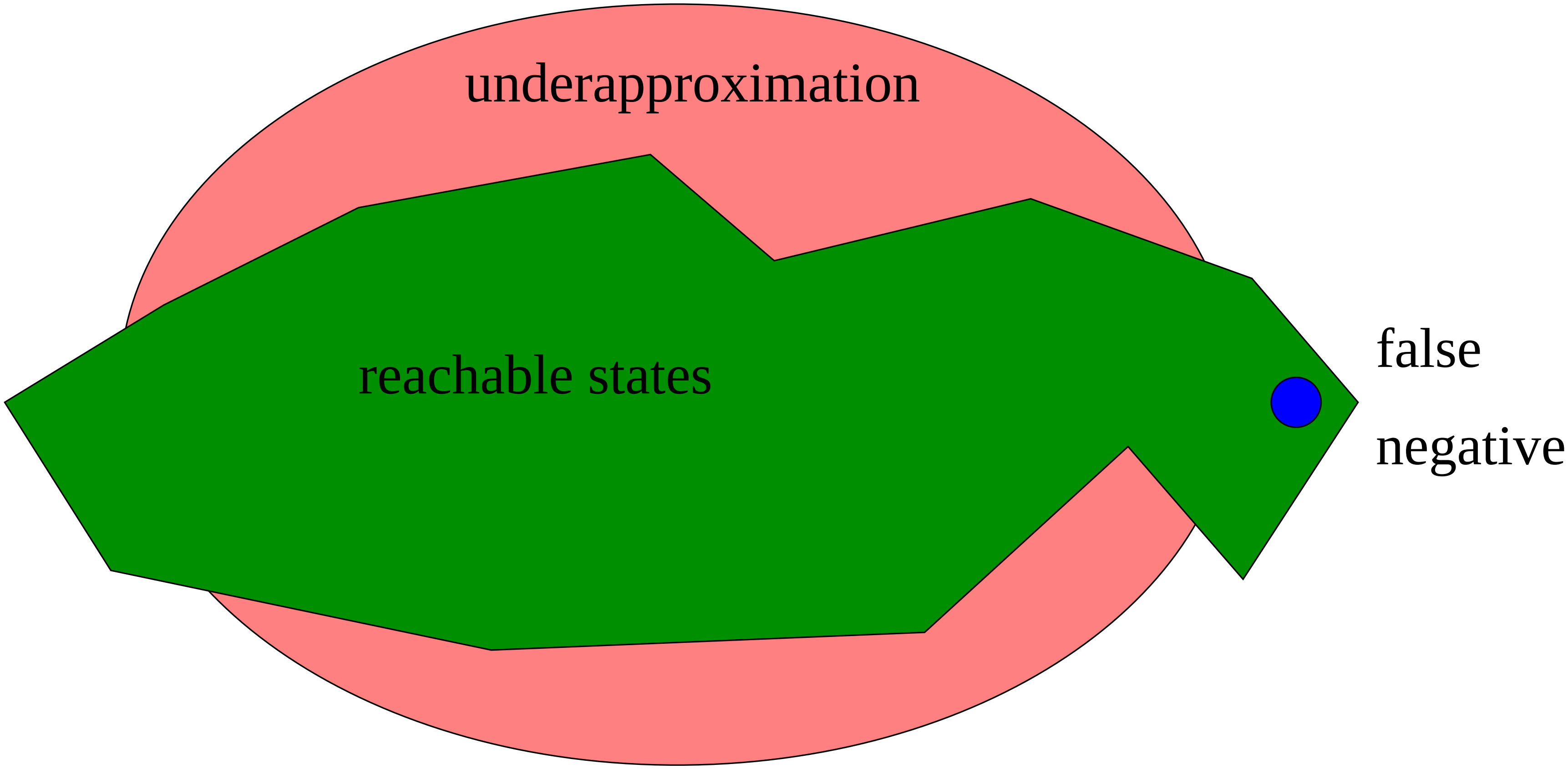
Sound approximations include all the behaviors and reachable states of the real system, but are easier to compute

But Sound Approximations Come with a Price



May flag an error that is unreachable in the real system: a false positive, or false alarm

Unsound Approximations Come with a Price, Too



Can miss real errors: a false negative

Predicate Abstraction

- The Zune example used data abstraction
  - A kind of abstract interpretation
- Replaces variables of complex data types by simpler (often finite) ones
  - e.g., integers replaced by {`negative`, `zero`, `positive`}
- But sometimes this doesn't work
  - Just replaces individual variables
  - Often its the relationship between variables that matters
- Predicate abstraction replaces some relationships (predicates) by Boolean variables

Another Example

```
start with r unlocked
do {
  lock(r)
  old = new
  if (*) {
    unlock(r)
    new++
  }
}
while old != new
want r to be locked at this point
unlock(r)
```

Abstracted Example

The significant relationship seems to be `old == new`

Replace this by `eq`, throw away `old` and `new`

```
[!locked]
do {
  lock(r)         [locked]
  eq = true       [locked, eq]
  if (*) {
    unlock(r)   [!locked, eq]
    eq = false  [!locked, !eq]
  }
}  [locked, eq] or [!locked, !eq]
while not eq
[locked, eq]
unlock(r)
```

Yet Another Example

```
z := n; x := 0; y := 0;
while (z > 0) {
  if (*) {
     x := x+1;
     z := z-1;
  } else {
     y := y+1;
     z := z-1;
  }
}
want y!= 0, given x != z, n > 0
```

- The invariant needed is `x + y + z = n`
- But neither this nor its fragments appear in the program or the desired property

Let's Just Go Ahead

First time into the loop

```
[n > 0]
z := n; x := 0; y := 0;
while (z > 0) {    [x = 0, y = 0, z = n]
  if (*) {
      x := x+1;
      z := z-1;    [x = 1, y = 0, z = n-1]
  } else {
      y := y+1;
      z := z-1;    [x = 0, y = 1, z = n-1]
  } [x = 1, y = 0, z = n-1] or [x = 0, y = 1, z = n-1]
}
```

Next time around the loop we'll have 4 disjuncts, then 8, then 16, and so on

This won't get us anywhere useful

Widening the Abstraction

- We could try eliminate disjuncts
- Look for a conjunction that is implied by each of the disjuncts
- One such is [x+y = 1, z = n-1]
- Then we'd need to do the same thing with [x+y = 1, z = n-1] or [x = 0, y = 0, z = n]
- That gives [x + y + z = n]
- There are techniques that can do this automatically
- This is where a lot of the research action is

Tradeoffs

- We're trying to guarantee absence of errors in a certain class
- Equivalently, trying to verify properties of a certain class
- Terminology is in terms of finding errors

  TP True Positive: found a real error

  FP False Positive: false alarm

  TN True Negative: no error, no alarm—OK

  FN False Negative: missed error

- Then we have

  Sound: no false negatives

  Recall: `TP/(TP+FN)` measures how (un)sound

  `TP+FN` is number of real errors

  Complete: no false alarms

  Precision: `TP/(TP+FP)` measures how (in)complete

  `TP+FP` is number of alarms

Tradeoff Space

- Basic tradeoff is between soundness and completeness
- For assurance, we need soundness
  - When told there are no errors, there must be none

  So have to accept false alarms
- But the main market for static analysis is bug finding in general-purpose software, where they aim merely to reduce the number of bugs, not to eliminate them
- Their general customers will not tolerate many false alarms, so tool vendors give up soundness
- Will consider the implications later
- Other tradeoffs are possible
  - Give up full automation: e.g., require user annotation

Tradeoffs In Practice

Testing is complete but unsound

Spark Ada with its Examiner is sound but not fully automatic

Abstract Interpretation (e.g., PolySpace) is sound but incomplete, and may not terminate

- Astrée is pragmatically complete for its domain

Pattern matchers (e.g. Lint, Findbugs) are not based on semantics of program execution, neither sound nor complete

- But pragmatically effective for bug finding

Commercial tools (e.g., Coverity, Code Sonar, Fortify, KlocWork, LDRA) are neither sound nor complete

- Pragmatically effective
- Different tools use different methods, have different capabilities, make different tradeoffs

Properties Checked

- The properties checked are usually implicit
    - e.g., uninitialized variables, divide by zero (and other exceptions), null pointer dereference, buffer overrun
- Much of this is compensating for deficiencies of C and C++
    - Some tools support Ada, Java, not much for MBD
    - But Mathworks has Design Verifier for Simulink
- Some tools support user-specified checks, but. . .
- Some tools look at resources
    - e.g., memory leaks, locks (not freed, freed twice, use after free)
- Some (e.g., AbsInt) can do quantitative analysis
    - e.g., worst case execution time, maximum stack height

Real Software

- It's not enough to check individual programs
- Need information from calls to procedures, subroutines
  - Analyze each in isolation, then produce a procedure summary for use by others
- Need summaries for libraries, operating system calls
- Analyzer must integrate with the build process
- Must present the information in a useful and attractive way
- Much of the engineering in commercial tools goes here

So How Good Are Static Analyzers?

- Some tool licences forbid benchmarking
- Hard to get representative examples
- NIST SAMATE study compared several
  - Found all had strengths and weaknesses
  - Needed a combination to get comprehensive bug detection
- This was bug finding, not assurance
- Anecdotal evidence is they are very useful for general QA
- Need to be tuned to individual environment
- e.g., Astrée tuned to Airbus A380 SCADE-generated digital filters is sound and pragmatically complete
- There are papers by Raoul Jetley and others of FDA applying tools to medical device software

Possible Futures: Combination With Testing

- Automated test generation is getting pretty good
- Use a constraint solver to find a witness to the path predicate leading to a given state
    - e.g., counterexamples from (infinite) bounded model checking using SMT solvers
- So try to see if you can generate an explicit test case to manifest a real bug for each positive turned up by static analysis
- Throw away those you cannot manifest
- Aha! Next generation of tools do this

Automated Test Generation

Test Generation

- Observe that a counterexample to an assertion: "control cannot reach this point" is a structural test case
- So BMC can be used for automated test generation
- Actually, a customized combination of SMC and BMC works best
  - Use SMC to reach first control point, then use BMC to extend to further control points
  - Get long tests that probe deep into the system
  - Can add test purposes that constrain the kinds of tests generated
    - e.g., Change the gear input by 1 at every step
  - Easily built because checkers are scriptable (in Scheme)

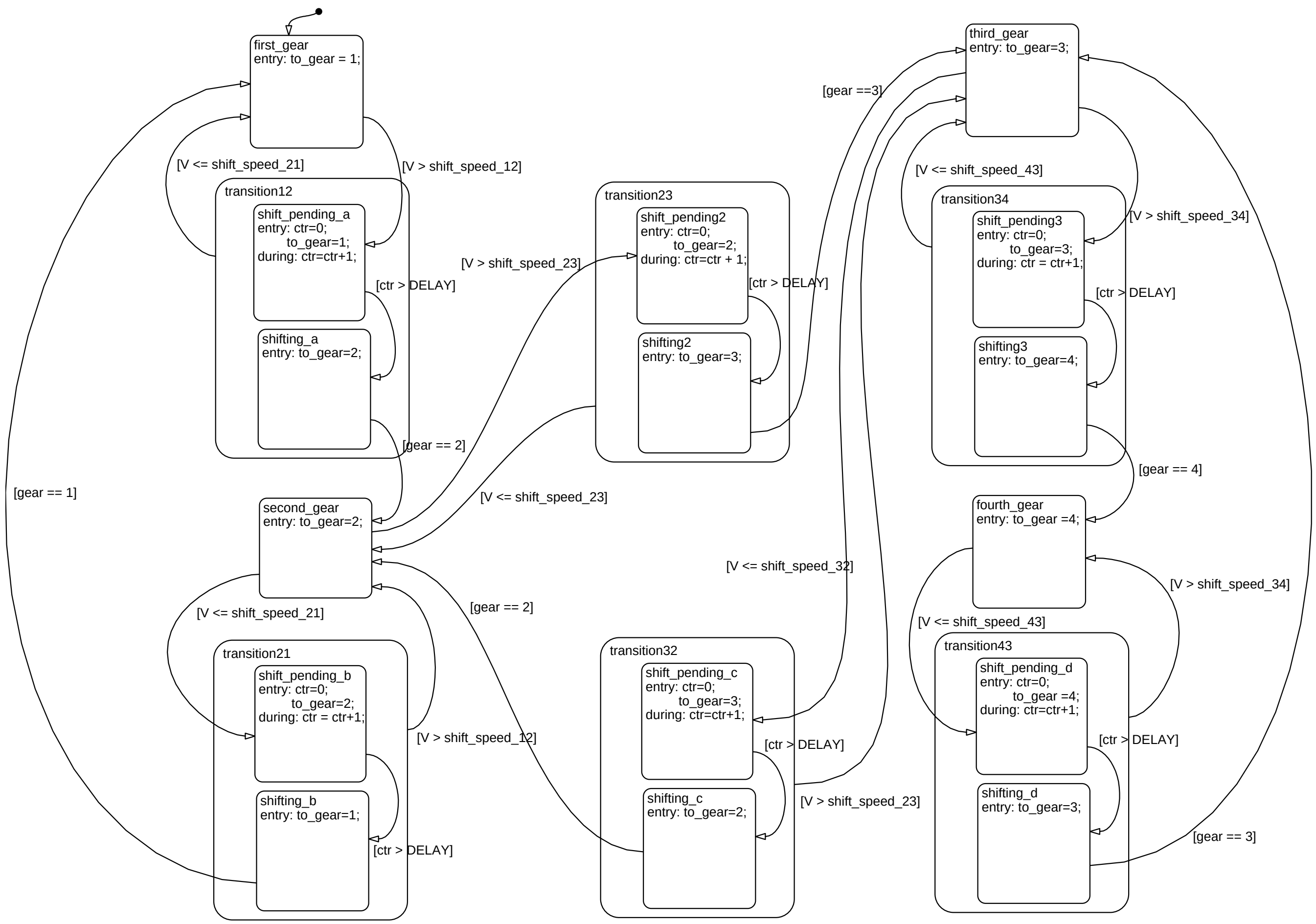
Core Of The SAL-ATG Test Generation Script

```
(define (extend-search module goal-list
         path scan prune innerslice start step stop)
   (let ((new-goal-list (if prune (goal-reduce scan goal-list path)
                           (minimal-goal-reduce scan goal-list path))))
    (cond ((null? new-goal-list) (cons '() path))
          ((> start stop) (cons new-goal-list path))
          (else
           (let* ((goal (list->goal new-goal-list module))
                  (mod (if innerslice
                          (sal-module/slice-for module goal) module))
                  (new-path
                   (let loop ((depth start))
                        (cond ((> depth stop) '())
                              ((sal-bmc/extend-path
                                 path mod goal depth 'ics))
                              (else (loop (+ depth step)))))))
             (if (pair? new-path)
                 (extend-search mod new-goal-list new-path scan
                                prune innerslice start step stop)
               (cons new-goal-list path)))))))
```

Outer Loop Of The SAL-ATG Test Generation Script

```
(define (iterative-search module goal-list
          scan prune slice innerslice bmcinit start step stop)
  (let* ((goal (list->goal goal-list module))
         (mod (if slice (sal-module/slice-for module goal) module))
         (path (if bmcinit
                   (sal-bmc/find-path-from-initial-state
                       mod goal bmcinit 'ics)
                 (sal-smc/find-path-from-initial-state mod goal))))
      (if path
          (extend-search mod goal-list path scan prune
              innerslice start step stop)
          #f)))
```

Example: Shift Scheduler in StateFlow



Demo: sal-atg -v 3 trans_ga monitored_system

trans_ga_goals.scm -ed 15 –incremental –testpurpose

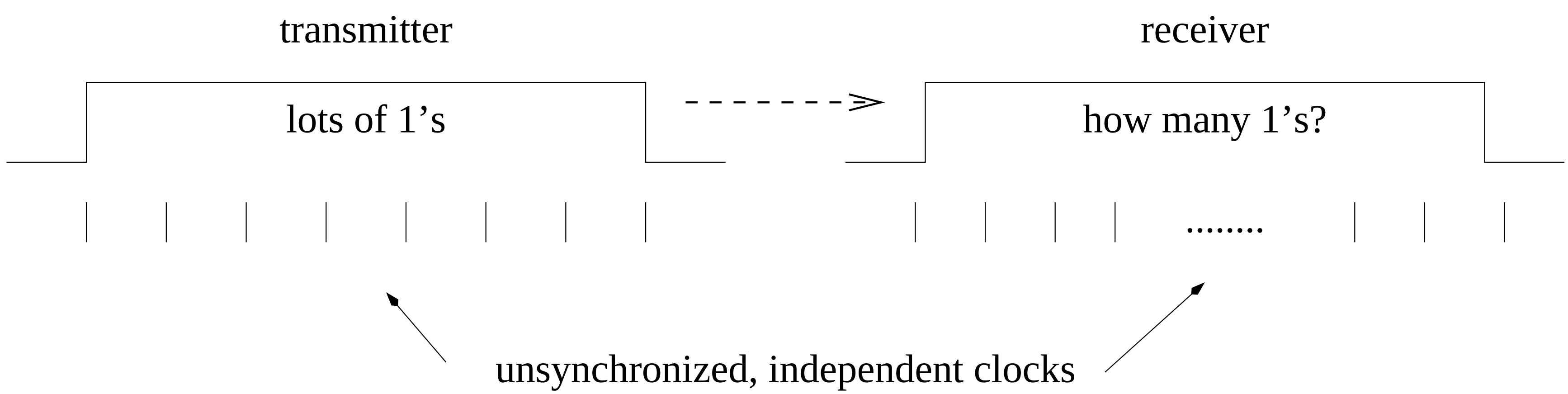
Timed Systems

Verification of Real Time Programs

- Continuous time excludes automation by finite state methods
- Timed automata methods (e.g., Uppaal)
  - Handle continuous time
  - But are defeated by the case explosion when (discrete) faults are considered as well
- SMT solvers can handle both dimensions
  - With discrete time, can have a clock module that advances time one tick at a time
    - ⋆ Each module sets a timeout, waits for the clock to reach that value, then does its thing, and repeats
  - Better: move the timeout to the clock module and let it advance time all the way to the next timeout
    - ⋆ These are Timeout Automata (Dutertre and Sorea): and they work for continuous time

Example: Biphase Mark Protocol

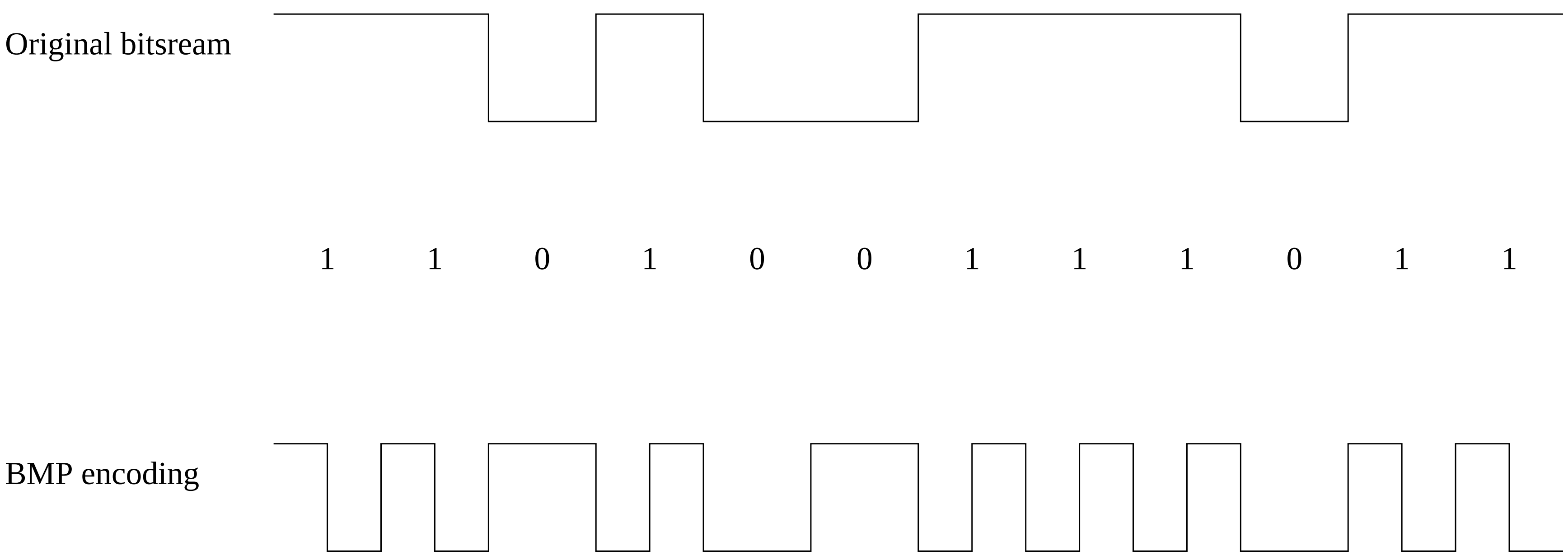
Biphase Mark is a protocol for asynchronous communication



- Clocks at either end may be skewed and have different rates, and jitter
- So have to encode a clock in the data stream
- Used in CDs, Ethernet
- Verification identifies parameter values for which data is reliably transmitted

Example: Biphase Mark Protocol (ctd)

- Flip the signal at the begining of every bit cell
- For a 1 bit, flip it in the middle, too
- For a 0 bit, leave it constant

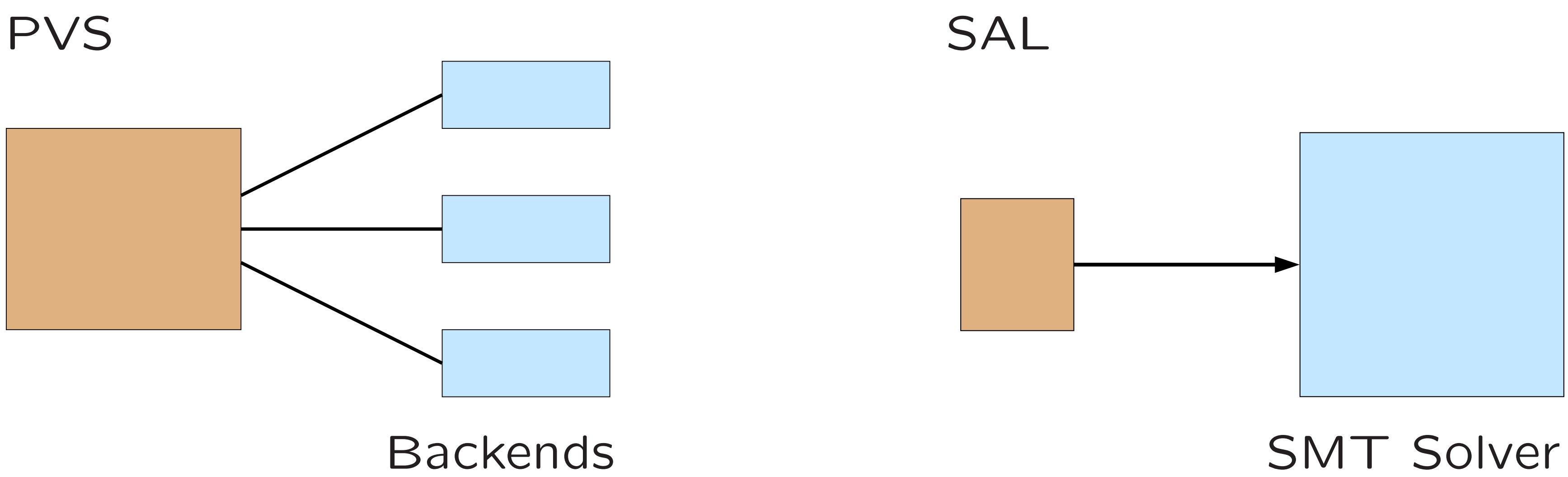


Prove this works provided the sender and reciever clocks run at similar rates

Biphase Mark Protocol Verification

- Verified by human-guided proof in ACL2 by J Moore (1994)
- Three different verifications used PVS
  - One by Groote and Vaandrager used PVS + UPPAAL required 37 invariants, 4,000 proof steps, hours of prover time to check
  - Also done by Dand Van Hung

Verification Systems vs. SMT-Based Model Checkers



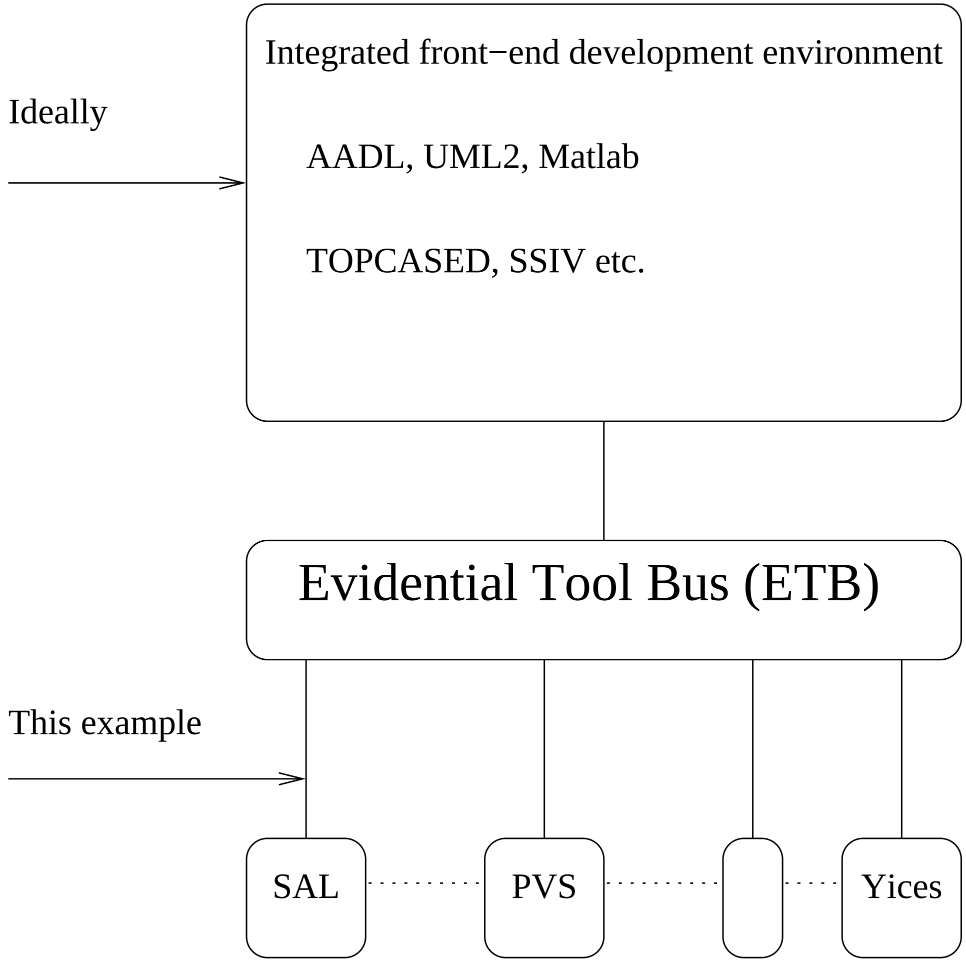
Actually, both kinds will coexist as part of the evidential tool bus—another talk

Biphase Mark Protocol Verification (ctd)

- Brown and Pike recently did it with sal-inf-bmc
  - Used timeout automata to model timed aspects
  - Statement of theorem discovered systematically using disjunctive invariants (7 disjuncts)
  - Three lemmas proved automatically with 1-induction,
  - Theorem proved automatically using 5-induction
  - Verification takes seconds to check
- Adapted verification to 8-N-1 protocol (used in UARTs)
  - Automated proofs more reusable than step-by-step ones
  - Additional lemma proved with 13-induction
  - Theorem proved with 3-induction (7 disjuncts)
  - Revealed a bug in published application note

Biphase Mark Protocol Verification (demo)

Ideally, use an integrated front end; here we look at raw model-checker input



Summary

- The challenge faced by formal methods analysis tools is how to search a huge space efficiently
- Theorem proving has developed efficient methods of local search (decision procedures etc.)
- Model checking showed that efficient global search was possible
- Now, methods are emerging that combine insights from both approaches in a promising way
- And there a pragmatic focus on finding methods for using the (still limited) capabilities of formal analysis tools to address useful, but partial issues in big, real systems
- I have never felt more optimistic about the prospects for formal analysis tools

Acknowledgments

- Very little of this is my work
- Most of it the work of my colleagues: Bruno Dutertre, Leonardo de Moura, Sam Owre, Shankar, Ashish Tiwari
- With valuable contributions from Saddek Bensalem, Pavol Cerny, Judy Crow, Jean-Christophe Filliâtre, Klaus Havelund, Friedrich von Henke, Yassine Lakhnech, Pat Lincoln, César Muñoz, Holger Pfeifer, Lee Pike, Harald Rueß, Vlad Rusu, Hassen Saïdi, Eli Singerman, Maria Sorea, Dave Stringer-Calvert, Mike Whalen, and many others

To Learn More

- Check out papers and technical reports at http://fm.csl.sri.com
- Information about PVS, and the system itself, is available from http://pvs.csl.sri.com
  - Version 4.0 is open source
  - Built in Allegro and CMU Lisp for Linux, Mac
- Yices: http://yices.csl.sri.com
  - Version 2.x is free binary
  - Written in C
  - Available as a library for C
- SAL: http://sal.csl.sri.com
  - Version 2.4 is open source
  - Written in Scheme, C++