Feasibility of Periodic Scan Schedules

Ants PI Meeting, Seattle, May 2000

Bruno Dutertre
System Design Laboratory
SRI International
e-mail: bruno@sdl.sri.com

Scan Scheduling

Scan scheduling:

- Given n hypothetical emitter types we can compute a priori a scan schedule
- There may be only a subset of these n emitters actually encountered during a mission
- The subset of relevant emitters may change as the mission progresses

Objective:

- Dynamically construct schedules, in real-time, on-line, using information about the emitters that are actually present

Central Issue:

- Given n emitters and their parameters, is there a schedule that satisfies the requirements? Is so find one.

Scan-Schedule Feasibility

Schedule Parameters:

- Whether in the static or dynamic case, we've assumed that a schedule is characterized by n dwell times (τ_i) and n revisit times (T_i), with

$$\sum_{i=1}^{n} \frac{\tau_i}{T_i} \leqslant 1.$$

Feasibility Issue:

- Given the parameters τ_i and T_i, can we construct a schedule such that the dwell intervals for different bands must not overlap?

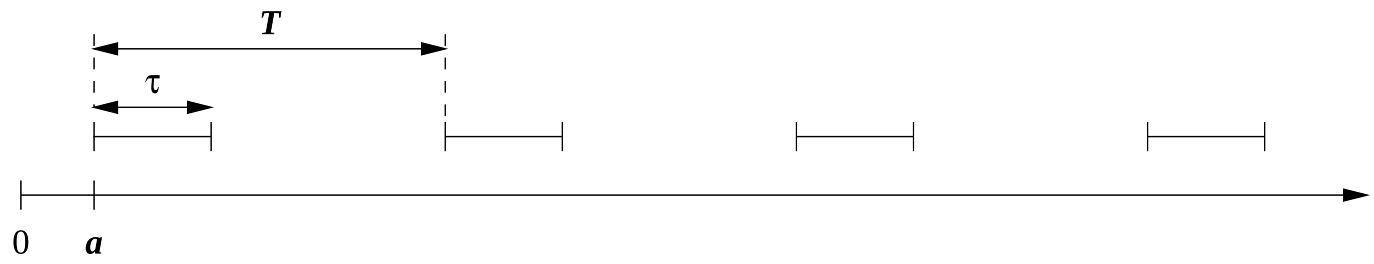
Problem:

- The condition above is necessary but not sufficient to ensure feasibility.

  For example, take $n = 3$, $\tau_1 = \tau_2 = \tau_3 = 1$ and $T_1 = 2$, $T_2 = 3$, $T_3 = 7$

Scan Schedule Parameters

- n disjoint frequency bands
- for each band: a triple (a_i, τ_i, T_i) such that $0 < \tau_i < T_i$ and $0 \leqslant a_i \leqslant T_i - \tau_i$



Schedule Construction

- Find $a_1, \ldots, a_n$ to ensure that dwell intervals for different frequency bands do not intersect.

Results on Scan-Schedule Feasibility

Theoretical complexity: the problem is NP-complete

Necessary condition: all the fractions T_i/T_j must be rational.

Case $n = 2$:

- The problem is equivalent to solving the system of inequalities

$$\begin{aligned}(a_2 - a_1) \bmod d &\geqslant \tau_1 \\ (a_1 - a_2) \bmod d &\geqslant \tau_2\end{aligned}$$

  where $d = \gcd(T_1, T_2)$.

- There is a solution and the schedule is feasible if and only if $\tau_1 + \tau_2 \leqslant d$.

Results on Scan-Schedule Feasibility (continued)

General case: $n \geqslant 3$

- We need to find $a_1, \ldots, a_n$ that satisfy two sets of constraints:

$$S_0 : \begin{cases} (a_1 - a_2) \bmod \gcd(T_1, T_2) \geqslant \tau_2 \\ \vdots \\ (a_n - a_{n-1}) \bmod \gcd(T_n, T_{n-1}) \geqslant \tau_{n-1} \end{cases}$$

$$S_1 : \begin{cases} 0 \leqslant a_1 \leqslant T_1 - \tau_1 \\ \vdots \\ 0 \leqslant a_n \leqslant T_n - \tau_n, \end{cases}$$

Results on Scan-Schedule Feasibility (continued)

Necessary conditions for feasibility:

$$\tau_i + \tau_j \leqslant d_{i,j}.$$

for $i = 1, \ldots, n$, $j = 1, \ldots, n$, and $i \neq j$.

Simplification:

- It is sufficient to look for solutions $(a_1, \ldots, a_n)$ such that

$$\begin{array}{rcccl} 0 & \leqslant & a_1 & < & 1 \\ 0 & \leqslant & a_2 & < & d_{1,2} \\ 0 & \leqslant & a_3 & < & \mathrm{lcm}(d_{1,3}, d_{2,3}) \\ & \vdots & & & \\ 0 & \leqslant & a_n & < & \mathrm{lcm}(d_{1,n}, \ldots, d_{n-1,n}). \end{array}$$

  where $d_{i,j} = \gcd(T_i, T_j)$

Resource Utilization

Sensor utilization:

$$U = \sum_{i=1}^{n} \frac{\tau_i}{T_i}$$

- This is the fraction of the time where the sensor does something useful, so we want U close to 1.
- Because of the constraints $\tau_i + \tau_j \leqslant d_{i,j}$, we have $\tau_i < d_{i,j}$ and $\tau_j < d_{i,j}$.
- U can then be very low since $d_{i,j}$ can be much smaller than T_i and T_j.
- This is confirmed by our first experiments.

Experiments

Algorithm Implemented:

- Depth-first search with backtracking.

Initial Experiments

- Randomly generated instances are rarely feasible (necessary conditions fail)
- For random instances constructed to satisfy the necessary conditions, the search algorithm is not practical
- Example:
  - n=60, all T_i are multiple of 100, $2000 \leqslant T_i \leqslant 3000$, and $0 \leqslant \tau_i \leqslant 20$.
  - Out of 100 random instances, 35 are feasible, 4 infeasible instances, 61 timeouts (6min CPU)
  - Average search time: 230s, average utilization: 0.25

Some Open Issues

Better Algorithms?

- Maybe by translation to integer programming

Special Instances

- High utilization can be achieved if the revisit times are harmonic (i.e., all are multiple of each other)
- but this is not a necessary condition, high U is possible under weaker conditions.

Bound on Achievable Utilization

- For a fixed set of revisit times, what is the maximal utilization one can get by varying the dwell times?

Conclusion

Using strictly periodic scan schedules is too restrictive:

- Feasibility and schedule construction are NP-complete
- Sensor utilization can be very low

More flexible schedules are needed:

- non-periodic schedules where the delay between successive dwells is not a constant ($T_i - \tau_i$) but can vary (also the length of dwell intervals can vary)
- for such schedules, we can solve all the feasibility issues by having a “feasible-by-construction” approach
- all we need is to extend the performance metrics (e.g. probability of detection or identification) to these non-periodic schedule. That’s a lot easier than solving feasibility problems.