On Location: Points about Regions

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8 October 1990

1 Introduction

In this paper we formalize Whitehead's construction for inducing point structures from region structures using a primitive relation of *connection* on regions [Whi79]. Our concern is to formulate a spatiotemporal analogue to the construction of temporal periods/points from events, and is reminiscent of the temporal constructions of Kamp [Kam79] and van Benthem [vBen83]. We compare our interpretation of Whitehead with the Kamp/van Benthem/Russell constructions and find some unresolved issues of interdefinability. Our goal is an apposite formulation of spatiotemporal locations as suggested for Situation Theory by Barwise and Perry [BP83].

1.1 Background

In [BP83, 51], Barwise and Perry take the individuation of spatiotemporal locations to be one of the basic uniformities across (real) situations. They define a collection of space-time locations, "connected 4-dimensional regions of time and space" [BP83, 51] as one of three primitives[1], along with three primitive relations: temporal precedence, temporal overlap, and spatial overlap. To motivate their choice of primitives, Barwise and Perry invoke a logical tradition originating with Whitehead [Whi79] and Russell [Rus88] in which events are primitive and "points" of space and "instants" of time are constructed out of events. They note that on the basis of Kamp's formalization of this construction, a given location could be identified with a set of pairs, $< p, t >$, where p is a spatial point and t is a temporal instant, and additional relations between locations such as spatial, temporal, and spatiotemporal inclusion could be defined [BP83, 51].[2] The revival of the Russell/Whitehead tradition invoked in [BP83] and exemplified in the Kamp and van Benthem

[1]The other two are individuals and relations. Hence *situation-types* are defined as the pair $< y, i >$, where $y =< r, x_1, \ldots, x_n >$ for a polarity i, n-ary relation r, and objects (i.e., individuals) $x_1, \ldots, x_n$, abstracting away from place and time. Locations enter in at the level of *states of affairs*, s, where $s =< l, s_0 >$ for a location l and situation-type s_0 [BP83, 53,55].

[2]It is interesting that Barwise and Perry define locations as spatiotemporal regions, but suggest that a given location can be defined as a set of pairs $< p, t >$ with distinct spatial and temporal elements. Our (re)construction

constructions [*op. cit.*] has provided an ontological basis for current work on the semantics of temporal phenomena. We attempt an analogous foundation for the spatiotemporal/spatial domain.[3] In keeping with this objective, we have chosen to follow Whitehead's construction because we prefer a formulation in terms of the intuitively spatial notion of *connectedness*, and the defined relation of *containment*. The discussion is organized as follows.

1.2 Organization

Assuming some familiarity with the temporal constructions proposed by Russell, van Benthem, Kamp, and others, we begin directly with a formalization of Whitehead's approach. We use the primitive notion *connectedness* of events and define regions from events. Whitehead's construction appears to encounter certain problems, including the generation of 'too many points'. We discuss this and related concerns before turning to the interdefinability of Whitehead's approach and that of Kamp/van Benthem. A brief final section suggests the usefulness of the 'region ontology' for analyzing spatial reference; we sketch an analysis of locative prepositional phrases exploiting the notion of spatial punctuality which follows from the proposed spatial construction.

2 Whitehead's Construction

In this section, we present a particular formalization of the somewhat informal construction offered in [Whi79]. Our construction differs from the original Whitehead proposal in certain respects which we point out in the course of the discussion. The reader may be somewhat surprised to encounter events rather than situations in the following presentation. We have included the discussion of event structures to facilitate comparison of the spatial and temporal ontologies, but the notion of event used here is generic and most of what we say below applies equally well to situations.[4]

At the level of events, the distinctions to be made are modest. We would like to be able to say that two events e_1 and e_2 are spatially *connected* in some primitive sense, i.e., that the spatial extensions of e_1 and e_2 are (in some unspecified way) 'in contact'. One type of *connection* is the derivative notion of inclusion, i.e., one event may spatially include another if they are connected as defined shortly. These simple spatial relations, one primitive, the other derived, make relatively few distinctions between events. In particular, differing events may behave identically with respect to these relations or, equivalently, events may be spatially indistinguishable relative to *connection* and *inclusion*. As in the temporal case, this indistinguishability provides a basis for mapping from event structures to the next level of structure, i.e., in the temporal domain, interval or period structures and in the spatial domain, region structures.

from Whitehead can be viewed as either spatiotemporal or purely spatial. For considerations in favor of a spatial interpretation, see [Cro90].

[3] Our approach is consistent with a brief remark by van Benthem in an appendix to [vBen83], where he mentions the possibility of a spatial "body ontology" along the lines of his temporal period structures.

[4] Of course, situations and events are potentially very different sorts of things, and we are not suggesting otherwise. *Cf.*, e.g., [AB85].

2.1 Events → Regions

To begin, we define an event structure $\mathcal{E}$.

1. $\mathcal{E} =< E, \mathcal{K} >$, where E is a set of events and $\mathcal{K}$ is the relation *connected*.

In particular, $\mathcal{K}$ is a symmetric, irreflexive relation on E, as indicated in the following axioms. Note that $\mathcal{K}$ is not transitive.

2. (a) $\forall e, e' \in E(e\mathcal{K}e' \rightarrow e'\mathcal{K}e)$.
   (b) $\forall e \in E\ \neg(e\mathcal{K}e)$.

To get an intuitive feel for connection, consider the following diagrams reproduced from Whitehead, illustrating various relations of connection [Whi79, 295]. The figures in these diagrams should be interpreted three-dimensionally, avoiding as much as possible the two-dimensional bias of the written page. Anticipating definitions that follow shortly, diagram (a) illustrates non-tangential inclusion, (b) and (c) tangential inclusion, and (e) and (f) external connection.

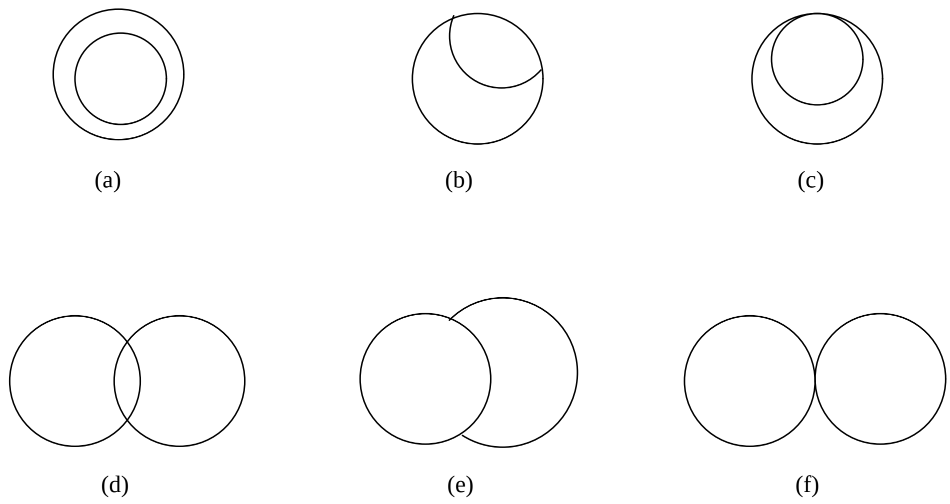

Figure 1: Various Flavors of Connection

Given this characterization of $\mathcal{K}$, the relation $\sqsubseteq_s$ of (spatial) inclusion can be defined as follows.

3. $\forall e, e' \in E, e \sqsubseteq_s e'$ iff $\forall e''(e''\mathcal{K}e \rightarrow e''\mathcal{K}e')$.

$\sqsubseteq_s$ is a preorder, i.e., a transitive, reflexive relation[5]. Symbolically,

[5]Whitehead's relation of inclusion is irreflexive and asymmetric. To arrive at an appropriate equivalence relation on E, $\sqsubseteq_s$ must be reflexive. Actually, we think reflexivity can be argued either way, perhaps equally arbitrarily. In the

4. (a) $\forall e, e', e'' \in E, e \sqsubseteq_s e' \sqsubseteq_s e'' \rightarrow e \sqsubseteq_s e''$.
   (b) $\forall e \in E, e \sqsubseteq_s e$.

Additionally, as defined above, $\sqsubseteq_s$ in concert with $\mathcal{K}$ also exhibits monotonicity. Given that $\mathcal{K}$ is symmetric, we have opted to state this axiom as a conjunction, rather than as a pair of left, right axioms typical of monotonicity statements for precedence and inclusion in the temporal domain. We use $\sqsupseteq_s$ to represent the converse relation to $\sqsubseteq_s$.

5. $\forall e', e'' \in E(e'\mathcal{K}e'' \rightarrow \forall e \sqsupseteq_s e'(e\mathcal{K}e'') \wedge \forall e \sqsupseteq_s e''(e\mathcal{K}e'))$.

Intuitively, spatial monotonicity asserts that superevents of an event e are *connected* to the same events to which e is *connected*.

As in the temporal case for $\sqsubseteq$, since $\sqsubseteq_s$ is neither asymmetric nor anti-symmetric on E, it follows that mutual inclusion can be used to partition E into classes of mutually included events. While such events are not identical, they are spatially indistinguishable. And again by analogy to the temporal construction, this accommodates a mapping from the event structure $\mathcal{E}$ into a *quotient region structure* $\mathcal{R}$ defined as follows.

6. $\mathcal{R} =< R, \sqsubseteq_s', \mathcal{K}' >$, where R is a set of all equivalence classes of mutual spatial inclusion, and $\sqsubseteq_s'$ and $\mathcal{K}'$ are induced relations of *inclusion* and *connection*, respectively.

We now consider possible definitions of the induced relations $\sqsubseteq_s'$ and $\mathcal{K}'$, beginning first with the properties of the induced relation $\mathcal{K}'$. There are at least two possible definitions of $\mathcal{K}'$, a weak and strong one, corresponding to whether *all* or only *some* points in an equivalence class have to be $\mathcal{K}$-related. Let $\overline{x}$ and $\overline{y}$ be regions, i.e., equivalence classes of mutually included events.

7. (a) Weak-$\mathcal{K}'$: $\overline{x}\ \mathcal{K}'\ \overline{y}$ iff $\exists w \in \overline{x}, \exists z \in \overline{y}, w\mathcal{K}z$.
   (b) Strong-$\mathcal{K}'$: $\overline{x}\ \mathcal{K}'\ \overline{y}$ iff $\forall w \in \overline{x}, \forall z \in \overline{y}, w\mathcal{K}z$.

The symmetry of both weak-$\mathcal{K}'$ and strong-$\mathcal{K}'$ follow immediately from the symmetry of $\mathcal{K}$. The irreflexivity of both weak- and strong-$\mathcal{K}'$ follow from the fact that no two equivalent events are connected. In particular,

8. let e, e' be in the same equivalence class, i.e., $e \sqsubseteq_s e' \sqsubseteq_s e$, and suppose $e\mathcal{K}e'$. Then by the second conjunct of the definition in 5 (with $e = e'$), $e\mathcal{K}e' \wedge e \sqsubseteq_s e' \rightarrow e'\mathcal{K}e'$. But $\mathcal{K}$ is irreflexive, hence e and e' can not be related by $\mathcal{K}$ and it follows that no two equivalent events are connected.

presence of equality, reflexivity and irreflexivity are trivially interdefinable; exchanging asymmetry for anti-symmetry entails reflexivity. And of course transitivity remains in either case. On the other hand, $\sqsubseteq_s$ can't be asymmetric if we intend to establish an equivalence relation on E to induce a quotient structure for regions. At the level of events, it seems reasonable to assume that (nonidentical) events are spatially indistinguishable (i.e., mutual inclusion should not be precluded), so we have chosen to part company with Whitehead on this point.

Thus both weak and strong $\mathcal{K}'$ are connectedness relations.[6]

The question of the induced relation $\sqsubseteq_s'$ is more straightforward, since the weak and strong definitions are provably equivalent. The transitivity of weak-$\sqsubseteq_s'$ and strong-$\sqsubseteq_s'$ follows trivially from the transitivity of $\sqsubseteq'$. Similarly, both weak- and strong-$\sqsubseteq_s'$ are reflexive. The anti-symmetry of weak-$\sqsubseteq_s'$ and strong-$\sqsubseteq_s'$ is an immediate consequence of the definition of the region structure $\mathcal{R}$ (*cf.* Definition 6 above). Note that filtering the preorder $\sqsubseteq_s$ through mutual-$\sqsubseteq_s$ gives the partial order $\sqsubseteq_s'$.

With either definition of $\mathcal{K}'$, it is easily verified that this mapping gives the following properties.

9. (a) $\mathcal{K}'$ is symmetric and irreflexive;
   (b) $\sqsubseteq_s'$ is transitive, reflexive and anti-symmetric;
   (c) $\mathcal{K}'$ is monotone in $\sqsubseteq_s'$;
   (d) $\forall x, y \in \overline{E}\ (x\mathcal{K}'y \rightarrow \forall z \sqsupseteq_s' x\ (z\mathcal{K}'y) \wedge \forall z \sqsupseteq_s' y\ (z\mathcal{K}'x))$;
   (e) $\forall \overline{e}, \overline{e}' \in \mathcal{R},\ \overline{e} \sqsubseteq_s' \overline{e}'$ iff $\forall \overline{z}\ (\overline{z}\mathcal{K}'\overline{e} \rightarrow \overline{z}\mathcal{K}'\overline{e}')$.

2.2 Regions $\rightarrow$ Points

The real work is done in the mapping from regions to points. Whitehead's idea is to define families of descending filters of regions, which he refers to as *groups of abstractive sets*. Points are then defined as families of "equal" filters, i.e., groups of abstractive sets which converge toward the same limit. The construction requires several additional notions, beginning with the following relations. Let R be a set of regions and $\mathcal{K}'$ and $\sqsubseteq_s'$ relations on R as defined above.

10. Spatial Overlap: $\forall r', r'' \in R,\ r' \bigcirc_s r''$ iff $\exists r \in R(r \sqsubseteq_s' r' \wedge r \sqsubseteq_s' r'')$.

11. External Connection: $\forall r, r' \in R,\ r\mathcal{K}^e r'$ iff $r\mathcal{K}'r' \wedge \neg(r \bigcirc_s r')$.

12. Tangential Inclusion: $\forall r, r' \in R,\ r \sqsubseteq_s^{\tan} r'$ iff
    $r \sqsubseteq_s' r' \wedge \exists r'' \in R(r''\mathcal{K}^e r \wedge r''\mathcal{K}^e r')$.

13. Non-Tangential Inclusion: $\forall r, r' \in R,\ r \sqsubseteq_s^{\not{t}\mathrm{an}} r'$ iff
    $r \sqsubseteq_s' r' \wedge \neg(\exists r'' \in R(r''\mathcal{K}^e r \wedge r''\mathcal{K}^e r'))$.

These definitions provide a way of characterizing Whitehead's notion of filters in this domain as sets of descending regions, i.e., sets of nested regions converging downward toward ever smaller regions. We might attempt to formalize this notion as follows:

[6] The weak and strong definitions correspond to the use of universal quantification and existential quantification. An interesting question is whether one could substitute any generalized quantifier over the pair of arguments, and still obtain a definition which satisfies the required properties of a connectedness relation. Of course a further question is the usefulness of generalized quantifiers in this context.

14. a filter F is a set of regions such that

    - $\forall r, r' \in F, r \neq r' \rightarrow (r \sqsubseteq_s^{\not{t}an} r' \vee r' \sqsubseteq_s^{\not{t}an} r)$,
    - $\neg(\exists r \in F \; \forall r' \in F \; (r \sqsubseteq_s' r')$.

The first clause of this definition specifies the nesting property and the second asserts that there is no region included in every member of the set F. We believe there are technical problems with this definition, particularly in relating the definition to the more common Russell/Kamp/van Benthem construction[7]. However, we also believe that this definition is true to Whitehead's original intent and therefore deserves further consideration. We continue by defining *equivalent* filters and *spatial point structures.*

Two filters are 'equivalent'[8], symbolized $F_1 \bowtie F_2$ if

15. $\forall r \in F_1 \; \exists r' \in F_2 (r' \sqsubseteq_s' r) \wedge \forall r \in F_2 \; \exists r' \in F_1 (r' \sqsubseteq_s' r)$.

In sum, two filters F_1 and F_2 are equivalent if every member of F_1 includes at least one member of F_2 and similarly, every member of F_2 includes at least one member of F_1. Accordingly, let R_1 be a member of F_1 and R_1' be a member of F_2. Then there is some member of F_2, say R_1' which is included in R_1; furthermore, there is some member of F_1, say R_2 which is included in R_1'; some member of F_2, say R_2' included in R_2 and so on as these sets of regions converge downward.

Not surprisingly, $\bowtie$ is an equivalence relation on filters, partitioning the set of filters into classes sharing a common convergence. Just as we previously used an equivalence relation to characterize regions in terms of events, following Whitehead we now propose to characterize (spatial) points as groups of abstractive sets i.e., $\bowtie$-equivalence classes of filters of regions. Actually, Whitehead further stipulates that a point is a group of (equivalent) abstractive sets with no (other) group of (equivalent) abstractive sets *incident* in it, where F_1 is *incident* in F_2 if $\forall r \in F_1, r' \in F_2 \; (r \sqsubseteq_s' r' \wedge F_1 \neq F_2)$.[9] It is clearly desirable to assure that the filters "in a point" converge toward the smallest possible limit. Accordingly, a spatial point structure, $\mathcal{P}$, is defined as

16. $\mathcal{P} =< S, \mathcal{K}_{\mathcal{P}}, \wp >$, where

    - S is the set of all equivalence classes of non-empty 'maximal' filters on $\mathcal{R}$,
    - $F_1 \mathcal{K}_{\mathcal{P}} F_2$ iff $\exists r_1 \in F_1, r_2 \in F_2$ such that $r_1 \mathcal{K}' r_2$,
    - $\wp$ is the set of all sets $f(r)$ of the form $f(r) = \{F \in S | r \in F\}$,
    - $f(r_1)\mathcal{K}'' f(r_2)$ iff $\exists F_1 \in f(r_1), F_2 \in f(r_2)$ such that $F_1 \mathcal{K}_{\mathcal{P}} F_2$.

[7] *Cf.* the discussion in Section 3.

[8] I.e., 'Equivalent' (only) with respect to their convergence.

[9] This constraint is motivated by concerns analogous to those which led van Benthem to the use of 'maximal' filters, and Kamp to 'maximal sets of pairwise overlapping events'.

It is readily apparent that the structure induced at this level is minimal; $\mathcal{K}_{\mathcal{P}}$ retains its characteristic properties, i.e., it is symmetric and irreflexive. True to our intuitions about the spatial domain, there is no notion of transitivity here, hence no pretense of a partial order on spatial 'points'.

As in the temporal case, if this construction is to succeed, $\mathcal{P}$ must be a spatial point structure whose induced region structure $\mathbf{R}(\mathcal{P})$ is isomorphic to $\mathcal{R}$ through the mapping f. Recall that $\mathbf{R}(\mathcal{P}) = < \wp, \mathcal{K}_{\mathcal{P}}, \subseteq >$. The relevant steps are as follows.

17. f is an isomorphism between $\mathcal{R}$ and $\mathbf{R}(\mathcal{P})$.
    Let the filter $F_r =_{def} \{r' \in R | r \sqsubseteq_s^{\not t an} r' \vee r' \sqsubseteq_s^{\not t an} r\}$, and $f(r)$ be as defined in (16).

    - f is surjective (onto): This follows from the definition of $\mathbf{R}(\mathcal{P})$.
    - f is injective (1-1).
    - f preserves $\mathcal{K}'$, (i.e., $r_1 \mathcal{K}' r_2$ only if $f(r_1)\mathcal{K}'' f(r_2)$).
    - f preserves $\sqsubseteq_s'$, (i.e., $r_1 \sqsubseteq_s' r_2$ only if $f(r_1) \subseteq f(r_2)$).
    - f anti-preserves $\mathcal{K}'$, (i.e., $f(r_1)\mathcal{K}'' f(r_2)$ only if $r_1 \mathcal{K}' r_2$).
    - f anti-preserves $\sqsubseteq_s'$, (i.e., $f(r_1) \subseteq f(r_2)$ only if $r_1 \sqsubseteq_s' r_2$).

2.3 Concerns With Filters

The preceding construction raises several issues, including the following.

18. (a) There may be too many maximal nested filters under non-tangential inclusion, which may generate 'too many points'.[10] We discuss this issue below.
    (b) There may be no easy way of defining maximal filters under non-tangential inclusion without invoking a strong choice principle (Zorn's Lemma). However, this is also the case for the more familiar construction of Russell/Kamp/van Benthem. As an antidote, van Benthem introduces the possibility of defining points with filters alone, without the maximality condition. Alternatively, a suitable definition of equivalence of filters might get around the problem with 'too many points', which can occur even without the maximality requirement.
    (c) Without invoking strong choice principles, there is no easy way to interdefine our reconstruction of Whitehead with the constructions used by Russell/Kamp/vanBenthem. We address this point in Section 3.

For now, we would like to return to issue (18a) and an example of one-dimensional regions (i.e., intervals) for which our Whitehead-style construction gives 'too many points'. Consider the two sets of rational intervals

19. • $M = \{\langle -r, +r \rangle : r > 0\}$

[10] *Cf.* [vBen83] for a similar discussion.

- $N = \{\langle -r, +1.2r\rangle : r > 0\}$

and let $MF(M), MF(N)$ be their extensions to maximal filters. These two sets appear to give different 'points', since neither set may contain any member of the other without violating the condition of non-tangential inclusion. Yet intuitively these sets should both define the point 0.

One potential solution is to consider a definition of equivalence of filters such as the following. Recall that $\mathcal{K}$ is symmetric, and let f and g be filters.

20. $f \sim g \Leftrightarrow (\forall r \in f)(\exists s \in g)(r\mathcal{K}s) \wedge (\forall s \in g)(\forall r \in f)(r\mathcal{K}s)$.

This definition is at variance with the one elaborated in (15) of Section 2.2, but allows the two sets of rational intervals in (19) above to define the same point if points are defined by *equivalence classes* of filters. Given the definition of $\sim$ and of points as equivalence classes of filters, it is possible to start from the intervals over the rationals[11], and obtain the rationals as the set of induced points.[12]

3 Relating the Constructions

In this section we explore the interdefinability of our reconstruction of Whitehead with the constructions of Russell/Kamp/van Benthem. To begin, if a Kampian universe satisfies a version of the *maximal subinterval principle* below, it is easy to relate Kamp's version of the filter construction with van Benthem's.[13] Using the maximal subinterval principle, van Benthemian filters can be constructed by iterating the operation N below. Similarly, using the operation O below, the mapping goes through in the other direction. Given this observation, we choose to use Kamp's version of the construction as maximal sets of overlapping intervals and speak in terms of *intervals*, which we take to be 'one-dimensional' regions. It should be clear that the discussion generalizes to arbitrary regions.

3.1 Overlapping to Nesting

Given Kamp's primitive of overlapping, how do we construct collections of nested intervals (i.e., pairwise mutually included intervals) from collections of mutually-overlapping intervals? The intent is that the 'point' that is constructed is the common 'part' of all the overlapping intervals[14]. One strategy is to identify an interval with each pair of overlapping intervals, such that the identified

[11] $INT(Q)$ in van Benthem's terminology.

[12] Since any two different rationals are separated, i.e., there is a positive distance between them, intervals in filters defining the two points must become separated as they get smaller, violating the previous condition of equivalence given in Section 2.2, but not the definition of $\sim$.

[13] Van Benthem uses the condition that intersections of any two intervals in the filter (i.e., their maximal common subinterval) must also be included in the filter. He also notes that this condition is logically stronger than needed - one only needs that *some* common subinterval is included, corresponding to our *subinterval principle*.

[14] Of course, there is no guaranteed common part in all the intended models, which is why the 'point' is identified with the actual collection.

interval also 'contains' the 'point'. Can we identify such an interval without invoking a principle of choice of controversial strength (and of mostly mathematical interest)?

The following *subinterval principle* is an example of an apparently uncontroversial principle facilitating the choice of this interval.

21. $(\forall x, y)(xOy \;\Rightarrow\; \exists z(z \sqsubseteq x \;\wedge\; z \sqsubseteq y)$.

The subinterval principle asserts that there is at least one common interval contained in two overlapping intervals. Kamp doesn't use the notion of $\sqsubseteq$ and there appears to be no uniform way to give a first-order implicit definition over all intended models from his axioms. As noted above, this is not the case for specific models. However, these specific models also satisfy the subinterval principle, as well as the stronger *maximal subinterval principle*

22. $(\forall x, y)(xOy \;\Rightarrow\; (\exists z)(z \sqsubseteq x \;\wedge\; z \sqsubseteq y \;\wedge\; (\forall w)(w \sqsubseteq x \;\wedge\; w \sqsubseteq y \Rightarrow w \sqsubseteq z)))$

which provides the choice principle for intervals considered above. It is easy to show that if there is a maximal subinterval as above, it is unique, and therefore in structures satisfying the maximal choice principle there is a way of associating an interval with overlapping subintervals, as we require for purposes of comparison.

Suppose $M(x, y)$ is the maximal subinterval associated with intervals x, y such that xOy is defined for overlapping pairs of intervals and is undefined otherwise. Suppose further that OI is a set of mutually overlapping intervals, i.e., $(\forall x, y \in OI)(xOy)$. Then

23. $N(OI) = \{M(x, y) : x, y \in OI\}$

is the collection of maximal subintervals of OI. There is no guarantee that $N(OI)$ is a collection of nested subintervals, and it seems one can only obtain such a collection from OI by repeated choice.

Let M_0 be $M(x, y)$ for some $x, y \in OI$. For the α'th stage of the construction, let $S_\alpha = \{x_0, \ldots. x_{2n}, x_{2n+1} \ldots : n < \alpha\}$ be the sequence of elements of OI used to define the first α M_i, and $N_\alpha = \bigcap_{n<\alpha} M_n$, i.e., the largest subinterval contained in all the M_n up to α. Now let $M_\alpha = M(x, y) \sqcap N_\alpha$ for some $x, y \in OI - S_\alpha$. Clearly this sequence of intervals produces a collection of nested intervals of cardinality the same as that of OI, assuming that OI is infinite and the sequence is well-defined. In order to produce such a sequence, one needs a *choice principle* over and above the *maximal subinterval principle* for well-ordering OI, and one needs an additional *extended maximal subinterval principle*, in order to take the maximal subinterval of a collection of α-many mutually overlapping intervals, for values of α up to the cardinality of OI. These are stronger choice principles than we might wish to assume in order to construct a collection of nested intervals 'corresponding' to a collection of overlapping intervals. Notice we did not require OI to be maximal. Supposing we wished to employ such principles to enable the construction. We might further wish to dispense with the maximal subinterval collection $N(OI)$ in favor of an arbitrary choice of common subinterval of x, y from the set $\{z : z \sqsubseteq x \;\wedge\; z \sqsubseteq y\}$, say to give $M'(x, y)$. Any choice principle strong enough to allow the construction as above would allow a simultaneous choice of $M'(x, y)$ for $x, y \in OI$, and would allow a similar 'arbitrary' choice of maximal included subinterval at stage α for each α. So it seems that if we were to employ this construction, both the maximal subinterval principle and the extended maximal subinterval principle can be eliminated in favor of the necessary, and far stronger, choice principle.

Another possibility for constructing a nested set of intervals from a set of of mutually overlapping intervals would be to iterate the application of the functor N to OI, i.e., to produce $N(OI), N(N(OI)), \ldots$ However, there seems to be no reason to expect this construction to reach a fixed point in which intervals are nested. For example, let RAT be the rational numbers, and consider the collection of intervals A over the rationals where we identify an interval with an ordered pair of ordered, unequal rationals given by the pairs

24. $\{\langle a, b\rangle : a, b \in RAT \ \wedge \ a < 0 \ \wedge \ b > 0\}$.

This collection would be a Kampian maximal set of overlapping intervals defining the point 0 in the model. A is a fixed point of the functor N, i.e., $N(A) = A$, and it is clear that the intervals are not nested.

There is no obvious way to define a set of nested intervals associated with a set of overlapping intervals, without invoking a principle such as a version of Well-Ordering Principle of suitable cardinality, along with an extended maximal subinterval principle. However, there is no requirement of a collection of nested intervals that the induced order given by the inclusion relation be a well-order[15], and our construction always produces a collection of nested intervals that is a well-order under the inclusion relation. Even if we were to make the strong choice assumptions needed, we should not expect that the construction is general because it doesn't give all nested interval constructions from overlapping-interval constructions. Moreover, it is even less likely to give a maximal nested-interval set unless a further assumption is made that all maximal sets of nested intervals are well-ordered and thus possible images of our construction — an assumption we would prefer to avoid.

3.2 Nesting to Overlapping

This situation may be contrasted with what pertains in the other direction from nested intervals to overlapping intervals. Suppose there were a *principle of the convex-hull*; for every pair x, y of intervals, there is a minimal interval $c(x, y)$ containing x, y, i.e.,

25. $(\forall x, y)(\exists c)(x \sqsubseteq c \ \wedge \ y \sqsubseteq c \ \wedge \ (\forall z)(x \sqsubseteq z \ \wedge \ y \sqsubseteq z \Rightarrow c \sqsubseteq z))$

and $c(x, y)$ is that c[16].

Let NI be a Whiteheadian maximal collection of nested intervals and consider

26. $O(NI) = \{c(x, y) : x, y \in NI\}$.

[15] It is easy to see that it is a linear order.

[16] Again, no choice required since c is unique.

It is easy to show that $O(NI)$ is a maximal collection of mutually overlapping intervals containing NI; in this direction the transformation is straightforward.

4 A Linguistic Application

In closing, we offer a glimpse of the linguistic utility of the construction proposed in Section 2 by considering the case of locative prepositional phrases. The examples cited below illustrate the role of a 'region ontology' in linguistic explanation; the issue is the explanatory leverage of an ontology in which points are not primitive, but derived. To begin, we need to reinforce our intuitions about the notion of 'punctuality' or 'atomicity' with respect to the preceding discussion.

Recall that a situation is assumed to be located 'at a given spatial point' just in case its location is a member of one of the filters in the set identified with that point. Accordingly, unless a location l either includes, or is included non-tangentially in two other locations l' and l'', where l' and l'' are not so related[17], l is spatially punctual in the sense of the preceding constructions. In other words, l is construed as spatially punctual because it participates in exactly one point in the point structure, i.e., there is only one set containing a filter F, such that l is a member of F. The implication, of course, is that a location is punctual (only) with respect to the discourse structure in which it participates, rather than in any absolute sense, geometric or otherwise.

To illustrate, consider the following two discourses, with locations l_i as indicated.

27.
- As agreed, Micah met Max in Davies Hall [l_1].
- They ate in a nearby cafe [l_2] and returned to the concert hall shortly after 8 o'clock [l_3].

28.
- Max went to a Kronos concert in Herbst Theater last night [l_1].
- Somewhat to his surprise, he sat two rows behind Micah [l_2].
- They talked at length in the lobby after the concert [l_3].

In (27), all three locations are spatially punctual, while in (28), only l_2 and l_3 are punctual. l_1 is non-atomic (i.e., not punctual) because it (non-tangentially) includes two events, namely l_2 and l_3 which are not related by inclusion, hence l_1 is 'divisible into', i.e., participates in, two distinct points.

Some readers may have noticed a slight peculiarity in example (28). It sounds more natural to say that *Max went to a Kronos concert* **at** *Herbst Theater*, which brings us to the main point of the section. We would like to propose that there is a primary sense of the preposition *at* that is inherently punctual. For example, we are suggesting that in (28) above, if we had used *at* instead of *in*, l_1 would have been atomic rather than non-atomic as given. More generally, we are suggesting that the constructions we have been discussing offer a plausible explanation for the intuition that

[17] I.e., it is not the case that $l' \sqsubseteq_s^{\not{t}an} l'' \vee l'' \sqsubseteq_s^{\not{t}an} l'$.

at frequently assumes a 'distant point of view'[18]. The explanation, of course, is that in such uses, the location associated with the preposition is seen as spatially punctual. Accordingly, in the example below, the first sentence may be followed by either (29b) or (29c), because the event is punctual, hence agnostic with respect to dimension and notions of internal/external. I.e., (29a) is non-committal as to whether Gerda and Ann met inside or outside the theater. On the other hand, if *at* is replaced by *in* in (29a), the first conjunct of (29b) is strange.

29. (a) Gerda met Ann at the theater.
    (b) They entered and went directly to their seats.
    (c) They spotted one another in the lobby.

5 Conclusion

We have formalized a construction based on Whitehead, which we presented as a spatial/spatiotemporal analogue of the temporal constructions proposed by Kamp and van Benthem. Our objective has been to provide an ontology particularly suited to the spatial/spatiotemporal domain. Quite naturally, this activity has raised issues of interdefinability between our Whitehead (re)construction and the constructions of Kamp and van Benthem. We have attempted to articulate these issues and in so doing, to provide a firmer ontological foundation for the notion of location in Situation Theory. Finally, we have illustrated the potential usefulness of this foundation with an example drawn from linguistics.

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[18] Related to this notion of remote viewpoint is the oft-noted fact that 'at' is tolerant with respect to the relation between objects/events. For example, in *Point b lies at the intersection of lines x and y*, the relation is exactly that of coincidence, while in *Eriks was at the garden gate* or *Ailey is at the beach*, the relation becomes increasingly vague.

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