

# Threatened by a Great Opportunity: Disruptive Innovation in Formal Verification

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## Since The Beginning. . .



- **Theorem provers** (or, at least, proof checkers) have been a central element in mechanized verification since the first systems of King (1969) and Good (1970)
- They've evolved over the years and become specialized for this application
- With decision procedures and other automation for arithmetic, data structures, recursively and inductively defined functions and relations etc.

Until. . .

- . . . the present?
- Most significant verifications were accomplished with a theorem prover (ACL2, HOL, Isabelle, PVS. . .)



## Then Along Came Model Checking

- Initially, these were just explicit state reachability analyzers
- Then BDDs and CTL
- But still finite state
- So ad-hoc **downscaling** required
- OK for debugging, not verification



- Same for **static analysis**

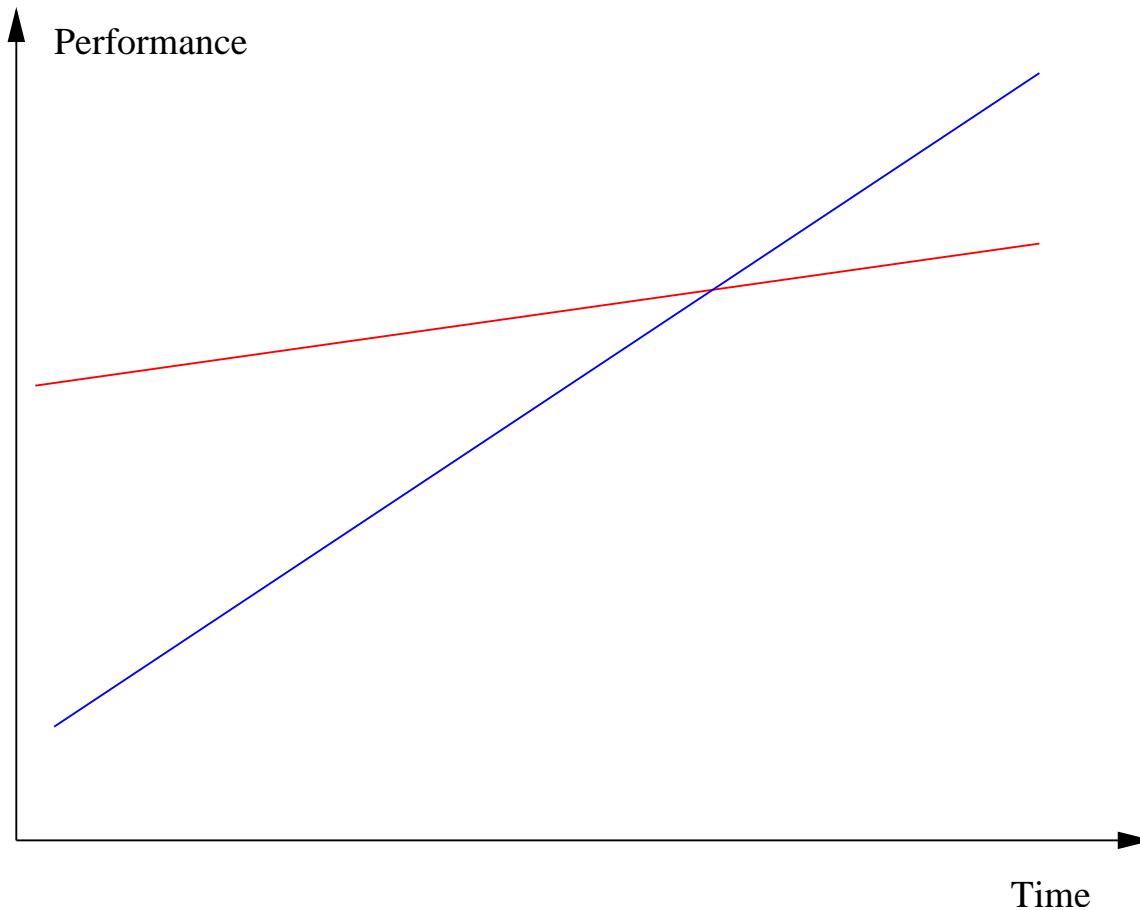
## ... And Automated Abstraction

- **Predicate abstraction** provides an automatable way to construct property preserving abstractions
- And spurious counterexamples can be mined to refine inadequate abstractions (CEGAR)



- Model checking starts to encroach on the space occupied by theorem proving

# Disruptive Innovation



Low-end disruption is when low-end technology overtakes the performance of high-end (Christensen)

## Incumbent's Response to Disruptive Innovation

- Try to **incorporate** the new technology
  - Theorem provers with model checking backends (e.g., PVS 1995)
  - Hard to stay current, architecture is too conservative
- Do your own disruption
  - **Extract** a technology that has disruptive performance
- Disrupt the disruptors
  - **Change** the rules of the game

## Do Your Own Disruption: SMT Solvers

- SMT stands for Satisfiability Modulo Theories
- SMT solvers extend decision procedures with the ability to handle arbitrary propositional structure
  - Traditionally, case analysis is handled heuristically in the theorem prover front end
    - ★ Have to be careful to avoid case explosion
  - SMT solvers use the brute force of modern SAT solving
- Or, dually, they generalize SAT solving by adding the ability to handle arithmetic and other decidable theories
- Application to verification
  - Via bounded model checking and  $k$ -induction



## SAT Solving

- Find satisfying assignment to a propositional logic formula
- Formula can be represented as a set of clauses
  - In CNF: conjunction of disjunctions
  - Find an assignment of truth values to variable that makes at least one literal in each clause TRUE
  - Literal: an atomic proposition  $A$  or its negation  $\bar{A}$
- Example: given following 4 clauses
  - $A, B$
  - $C, D$
  - $E$
  - $\bar{A}, \bar{D}, \bar{E}$

One solution is  $A, C, E, \bar{D}$

( $A, D, E$  is not and cannot be extended to be one)

- Do this when there are 1,000,000s of variables and clauses

## SAT Solvers

- SAT solving is the quintessential NP-complete problem
- But **now amazingly fast in practice** (most of the time)
  - Breakthroughs (starting with Chaff) since 2001
    - ★ Building on earlier innovations in SATO, GRASP
  - Sustained improvements, honed by competition
- **Has become a commodity technology**
  - MiniSAT is 700 SLOC
- **Can think of it as massively effective search**
  - So **use it** when your problem can be formulated as SAT
- **Used in bounded model checking and in AI planning**
  - Routine to handle  $10^{300}$  states

## SAT Plus Theories

- SAT can encode operations and relations on **bounded** integers
  - Using bitvector representation
  - With adders etc. represented as Boolean circuits

And other **finite** data types and structures

- But cannot do not **unbounded** types (e.g., reals), or **infinite** structures (e.g., queues, lists)
- And even bounded arithmetic can be **slow** when large
- **There are fast decision procedures for these theories**
- But their basic form works only on **conjunctions**
- **General propositional structure requires case analysis**
  - Should use efficient search strategies of SAT solvers

**That's what an SMT solver does**

## Decidable Theories

- Many useful theories are **decidable**

(at least in their unquantified forms)

- **Equality** with **uninterpreted function symbols**

$$x = y \wedge f(f(f(x))) = f(x) \supset f(f(f(f(f(y)))))) = f(x)$$

- Function, record, and tuple **updates**

$$f \text{ with } [(x) := y](z) \stackrel{\text{def}}{=} \text{if } z = x \text{ then } y \text{ else } f(z)$$

- **Linear arithmetic** (over integers and rationals)

$$x \leq y \wedge x \leq 1 - y \wedge 2 \times x \geq 1 \supset 4 \times x = 2$$

- Special (fast) case: **difference logic**

$$x - y < c$$

- **Combinations** of decidable theories are (usually) decidable

*e.g.*,  $2 \times \text{car}(x) - 3 \times \text{cdr}(x) = f(\text{cdr}(x)) \supset$

$$f(\text{cons}(4 \times \text{car}(x) - 2 \times f(\text{cdr}(x)), y)) = f(\text{cons}(6 \times \text{cdr}(x), y))$$

Uses **equality**, **uninterpreted functions**, **linear arithmetic**, **lists**

## SMT Solving

- Individual and combined decision procedures decide **conjunctions** of formulas in their decided theories
- **SMT allows general propositional structure**
  - e.g.,  $(x \leq y \vee y = 5) \wedge (x < 0 \vee y \leq x) \wedge x \neq y$   
... possibly continued for 1000s of terms
- Should exploit search strategies of modern SAT solvers
- So replace the **terms** by **propositional variables**
  - i.e.,  $(A \vee B) \wedge (C \vee D) \wedge E$
- Get a **solution from a SAT solver** (if none, we are done)
  - e.g.,  $A, D, E$
- **Restore the interpretation of variables and send the conjunction to the core decision procedure**
  - i.e.,  $x \leq y \wedge y \leq x \wedge x \neq y$

## SMT Solving by “Lemmas On Demand”

- If satisfiable, we are **done**
- If not, ask SAT solver for a **new assignment**
- **But isn't it expensive to keep doing this?**
- Yes, so first, do a little bit of work to find fragments that **explain** the unsatisfiability, and send these back to the SAT solver as additional constraints (i.e., lemmas)
  - $A \wedge D \supset \bar{E}$  (equivalently,  $\bar{A} \vee \bar{D} \vee \bar{E}$ )
- Iterate to termination
  - e.g.,  $A, C, E, \bar{D}$
  - i.e.,  $x \leq y, x < 0, x \neq y, y \not\leq x$  (simplifies to  $x < y, x < 0$ )
  - A satisfying assignment is  $x = -3, y = 1$
- This is called “**lemmas on demand**” (de Moura, Rues, Sorea) or “DPLL(T)”; **it yields effective SMT solvers**

## Fast SMT Solvers

- There are several effective SMT solvers
  - Our **ICS** was among the first (released 2002)
    - ★ Precursors include CVC, LPSAT, Simplify...
  - Now replaced by **Yices** (1.0 released last week)
  - European examples: **Barcelologic**, **MathSAT**
- **Yices** decides formulas in the combined theories of: linear arithmetic over integers and reals (including mixed forms), fixed size bitvectors, equality with uninterpreted functions, recursive datatypes (such as lists and trees), extensional arrays, dependently typed tuples and records of all these, lambda expressions, and some quantified formulas
- **SMT solvers are being honed by competition**
  - Provoked by our benchmarking in 2004
  - Now institutionalized as part of CAV, FLoC

## SMT Competition

- Various divisions (depending on the theories considered)
  - Equality and uninterpreted functions
  - Difference logic ( $x - y < c$ )
  - Full linear arithmetic
    - ★ For integers as well as reals
  - Extensional arrays, bitvectors, quantification . . . etc.
- ICS was the most uniformly effective in 2004
- Yices 0.2 and Simplics (prototypes for Yices 1.0) won the advanced divisions in 2005, came second to Barcelogic in all the others
- Look out for the 2006 competition
  - The results are now in: Yices 1.0 won all 11 divisions



## Bounded Model Checking (BMC)

- Given system specified by initiality predicate  $I$  and transition relation  $T$  on states  $S$

- Is there a counterexample to property  $P$  in  $k$  steps or less?

- Find assignment to states  $s_0, \dots, s_k$  satisfying

$$I(s_0) \wedge T(s_0, s_1) \wedge T(s_1, s_2) \wedge \dots \wedge T(s_{k-1}, s_k) \wedge \neg(P(s_1) \wedge \dots \wedge P(s_k))$$

- Given a Boolean encoding of  $I$ ,  $T$ , and  $P$  (i.e., circuit), this is a **propositional satisfiability (SAT)** problem

- But if  $I$ ,  $T$  and  $P$  use decidable but unbounded types, then it's an **SMT** problem: **infinite bounded model checking**

- (Infinite) BMC also generates **test cases** and **plans**

- **State the goal as negated property**

$$I(s_0) \wedge T(s_0, s_1) \wedge T(s_1, s_2) \wedge \dots \wedge T(s_{k-1}, s_k) \wedge (G(s_1) \vee \dots \vee G(s_k))$$

## $k$ -Induction

- BMC extends from **refutation** to **verification** via  $k$ -induction
  - Other ways include finding diameter of the statespace, abstraction/refinement, using interpolants to find fixpoint

- Ordinary inductive invariance (for  $P$ ):

**Basis:**  $I(s_0) \supset P(s_0)$

**Step:**  $P(r_0) \wedge T(r_0, r_1) \supset P(r_1)$

- Extend to induction of depth  $k$ :

**Basis:** No counterexample of length  $k$  or less

**Step:**  $P(r_0) \wedge T(r_0, r_1) \wedge P(r_1) \wedge \dots \wedge P(r_{k-1}) \wedge T(r_{k-1}, r_k) \supset P(r_k)$

These are close relatives of the BMC formulas

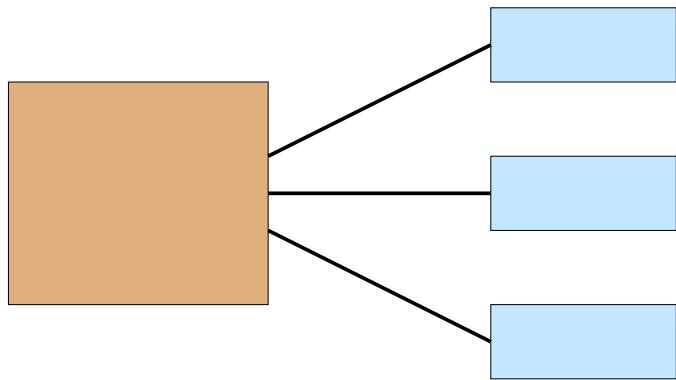
- Induction for  $k = 2, 3, 4 \dots$  may succeed where  $k = 1$  does not
- Note that counterexamples help debug invariant

## Evolution of SMT-Based Model Checkers

- Replace the backend decision procedures of a verification system with an SMT solver, and specialize and shrink the higher-level proof manager
- Example:
  - SAL language has a type system similar to PVS, but is specialized for specification of state machines  
(as finite- or infinite-state transition relations)
  - The SAL infinite-state bounded model checker uses an SMT solver (Yices), so handles specifications over reals and integers, uninterpreted functions, etc.
  - Often used as a model checker (i.e., for refutation)
  - But can perform verification with a single higher level proof rule:  $k$ -induction (with lemmas)

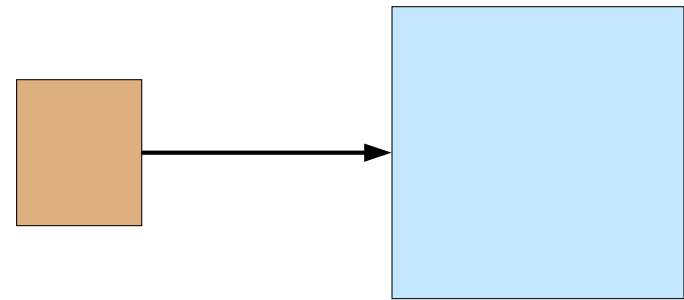
## Verification Systems vs. SMT-Based Model Checkers

PVS



Backends

SAL



SMT Solver

Actually, both kinds will coexist as part of the [evidential tool bus](#)—see later

## Application: Verification of Real Time Programs

- **Continuous time** excludes automation by finite state methods
- **Timed automata** methods handle continuous time
  - But are defeated by the **case explosion** when (discrete) faults are considered as well
- **SMT solvers can handle both dimensions**
  - With discrete time, can have a clock module that advances time one tick at a time
    - ★ Each module sets a timeout, waits for the clock to reach that value, then does its thing, and repeats
  - **Better**: move the timeout to the clock module and let it advance time all the way to the next timeout
    - ★ These are **Timeout Automata** (**Dutertre and Sorea**): and **they work for continuous time**

## Example: Biphase Mark Protocol

- **Biphase Mark** is a protocol for asynchronous communication
  - Clocks at either end may be **skewed** and have **different rates**, and **jitter**
  - So have to encode a clock in the data stream
  - Used in CDs, Ethernet
  - **Verification** identifies parameter values for which data is **reliably transmitted**
- **Verified** by human-guided proof in **ACL2** by J Moore (1994)
- **Three different verifications** used **PVS**
  - One by Groote and Vaandrager used **PVS + UPPAAL**
  - Required **37** invariants, **4,000** proof steps, **hours** of prover time to check

## Biphase Mark Protocol (ctd)

- Brown and Pike recently did it with `sal-inf-bmc`
  - Used **timeout automata** to model timed aspects
  - Statement of theorem discovered **systematically** using **disjunctive invariants** (7 disjuncts)
  - **Three** lemmas proved automatically with **1-induction**,
  - Theorem proved automatically using **5-induction**
  - Verification takes **seconds** to check
  - Demo:  

```
sal-inf-bmc -v 3 -d 5 -i -1 10 -1 11 -1 12 biphase t0
```
- **Adapted** verification to 8-N-1 protocol (used in UARTs)
  - **Automated proofs more reusable than step-by-step ones**
  - Additional lemma proved with **13-induction**
  - Theorem proved with **3-induction** (7 disjuncts)
  - **Revealed a bug** in published application note

## SMT Solvers as Disruptive Innovation

Disruptive to:

**SAT solvers:** anything a SAT solver can do, an SMT solver does better

**Constraint solvers:** maybe

**Ordinary BMC:** Infinite BMC is often faster than (finite) BMC (SMT solver on the real problem is faster than SAT on the bit-blasted representation)

**CEGAR loops:** SMT extends to MaxSMT and, dually, extraction of unsat cores

**Traditional theorem provers:** at the very least, they need an SMT solver for endgame proofs (PVS 4.0 uses Yices), but hard to rival the **perfect adaptation** of infinite BMC and  $k$ -induction

So...



## Disrupting The Disruptors

- Many modern tools are **highly specialized**
  - e.g., **SDV**, **Blast**, **Smallfoot**, **Terminator**, **Daikon**, **Astrée**, **Absint**
- They are each **perfectly adapted** to their niche
- **But what if we have a slightly different problem?**
- **Cannot get at the internals**
  - e.g., Blast computes predicate abstractions internally but you cannot extract the abstracted transition relation
- And **cannot integrate** different systems without a lot of coding
- So perhaps **open integration** is the thing to go for

## Integration Example: LAST

- **LAST** (Xia, DiVito, Muñoz) generates **MC/DC tests** for avionics code involving **nonlinear arithmetic** (with **floating point** numbers, **trigonometric** functions etc.)
- **Applied it to Boeing autopilot simulator**
  - Modules with upto 1,000 lines of C
  - 220 decisions
- **Generated tests to (almost) full MC/DC coverage in minutes**

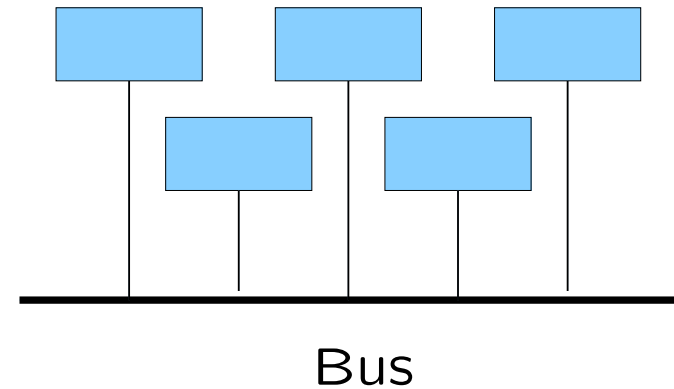
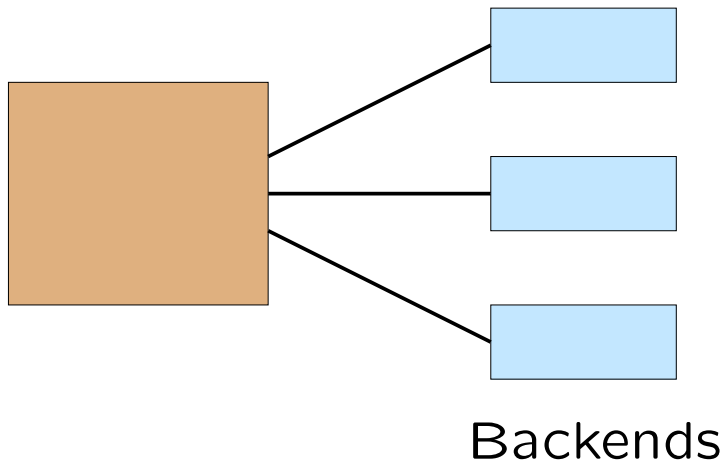
## Structure of LAST

- It's built on [Blast](#) (Henzinger et al)
  - A software model checker, itself built of components
  - Including CIL and CVC-Lite
- But extends it to handle nonlinear arithmetic using [RealPaver](#) (a numerical nonlinear constraint unsatisfiability checker)
  - Added 1,000 lines to [CIL](#) front end for MC/DC
  - Added 2,000 lines to [RealPaver](#) to integrate with [CVC-Lite](#) (Nelson-Oppen style)
  - Changed 2,000 lines in [Blast](#) to tie it all together

## Need To Integrate Heterogeneous Components

- Modern formal methods tools
    - And particularly their **components**often do something other than **verification**
  - They also do **refutation** (bug finding)
  - And **test-case generation**
  - And **controller synthesis**
  - And construction of **abstractions** and **abstract interpretations**
  - And generation of **invariants**
  - And ...
  - **Observe that these tools can return objects other than verification outcomes**
    - **Unsat cores, test cases, abstractions, invariants**
- Hence, **heterogeneous**

## Backends or Bus?



- Heterogeneity argues against theorem prover backends
- Bus is a federation of equals; theorem prover is just another component

## A Tool Bus

- Want to construct these customized combinations and integrations **easily and rapidly**
- The integrations are coarse-grained (hundreds, not millions of interactions per analysis), so they do not need to share state
- So we could take the outputs of one tool, massage it suitably and pass it to another and so on
- A combination of XML descriptions, translations, and a scripting language could probably do it
- Suitably engineered, we could call it a **tool bus**

## But ...

- But we'd need to know the names and capabilities of the tools out there and explicitly to script the desired interactions
  - And we'd be vulnerable to change
- Whereas I would like to exploit whatever is out there
  - And in 15 years time there may be lots of things out there
- Sounds a bit like Service Oriented Architecture
- Then we need an ontology etc.

## A Formal Tool Bus

- The data manipulated by tools on bus are formulas in **logic**
- **So logic can provide our ontology**
- Each tool operates on a **sublogic**
  - Syntactic differences masked with XML wrappers
- Open ended, but top logic at least as expressive as PVS
  - **Higher order, with predicate, structural, and dependent subtypes, abstract data types, recursive and inductive definitions, parameterized theories, interpretations**
- With structured **representations** for important cases
  - **State machines (as in SAL), counterexamples, process algebras, temporal logics ...**
  - Handled directly by some tools, can be expanded to underlying semantics for others



## Tool Bus Judgments

The tools on the bus evaluate and construct predicates over assertions in the logic—we call these **judgments**

**Parser:** A is the AST for string S

**Prettyprinter:** S is the concrete syntax for A

**Typechecker:** A is a well-typed formula

**Finiteness checker:** A is a formula over finite types

**Abstructor to PL:** A is a propositional abstraction for B

**Predicate abtractor:** A is an abstraction for formula B wrt. predicates  $\phi$

**DP:** A is satisfiable

**DP:** C is a context (state) representing input G

**SMT:**  $\rho$  is a satisfying assignment for A

## Tool Bus Queries

- Judgments include the name of the tool that decided its assertion:  $T \vdash A$

- And both tool and assertion can include **variables**

**Query:**  $? \vdash \text{predicate-abstraction}(?, B, \phi)$

**Response:**

$\text{SAL-abstractor}(\dots) \vdash \text{predicate-abstraction?}(A, B, \phi)$

The responding tool constructs the witness, and returns its **handle**  $A$ , along with its own **invocation**

- So tools operate by **implicit invocation**
- Tools **interact** through **forward** and **backward chaining** on queries

## Tool Bus Operation

- The tool bus operates like a distributed datalog framework, chaining on queries and responses
- Similar to SRI AIC's Open Agent Architecture
  - And maybe similar to MyGrid, Linda, TIB, ...?
- Can have hints, preferences etc.
- The bus needs to integrate with version management
- Tools can be local or remote
- Tools can run in parallel, in competition
- Some tools may be simple scripts

## Tool Bus Scripts

- Example
  - If A is a **finite** state machine and P a safety property, then a **model checker** can verify P for A
  - If B is a **conservative abstraction** of B, then verification of B verifies A
  - If A is a state machine, and B is **predicate abstraction** for A, then B is conservative for A
- How do we know this is **sound**?
- And that we can **trust the computations** performed by the components?

## An Evidential Tool Bus

- Each tool should deliver **evidence** for its judgments
  - Could be **proof objects** (independently checkable trail of basic deductions), or **hints** for a **reference implementation**
  - Could be **reputation** (“Proved by PVS”)
  - Could be **diversity** (“using both Yices and CVC-Lite”)
  - Could be **declaration** by user
    - ★ “Because I say so”
    - ★ “By testing”
- A full judgment is  $T \vdash E : A$ , which is the claim that **tool instance  $T$  provides evidence  $E$  for assertion  $A$**
- And the tool bus assembles these (on demand)
- Can chain on the evidence component
- To construct evidence for overall analysis for use in a safety or assurance case—hence **evidential** tool bus

## The Evidential Tool Bus

- There should be only one evidential tool bus
- Just like only one WWW
- How to do it?
  - Standards committee?
  - Competition and cooperation!
- We'll be building one
  - Initially to integrate PVS and SAL with invariant generators
  - And to reconstruct Hybrid-SAL
- But will not be specialized to our tools
- We'll appreciate your input, and hope you'll like to use it, and to attach your tools

## Summary

- Disruptive innovations are sweeping through our field
- We can both contribute to these and use them
- Contribution to disruption: **SMT solvers**
  - In addition to backend solvers and BMC, these can be used in **automated test generation**, **predicate abstraction**, **invariant generation and verif'n**, **(weighted) Maxsat/Mincore**, **extended static checking and extended type checking**, and **temporal/metric plan synthesis and verif'n**
- Harness disruption: **The Evidential Tool Bus**
  - Lightweight decentralized open-ended architecture for integrating heterogeneous components
  - We'll use it to reconstruct **Hybrid SAL**, construct and integrate many methods for **invariant generation**
- **The real targets for disruption are traditional methods for software development, validation, and certification**

## Thank you!

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- You can get our tools and papers from <http://fm.csl.sri.com>
- This one: <http://www.csl.sri.com/~rushby/abstracts/sefm06>