

SafeComp invited talk on 25 Sep 2013, substantially modified from  
NFM 2013 Keynote on 16 May 2013, shortened from  
Talk at NICTA, Sydney on 23 April 2013, slight change on  
“Distinguished Lecture” at Ames Iowa, 7 Mar 2013,  
based on Dagstuhl talk in January 2013, combined with  
Cambridge University talk in 2011, and “Distinguished Lecture”  
at Institute of Information Sciences, Academia Sinica, 14 Feb  
2011, which was based on LAW 2010 invited talk 7 December  
2010

# Logic and Epistemology in Safety Cases

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# Introduction

- System and software **safety** are our goals
- And **assurance** for these
- **Assurance Cases** provide a modern framework for doing this
- But I'm a **formal methods** guy
  - I believe in Leibniz' Dream (later)
- So I want to explore relationships between formal methods
  - Actually, **formal verification** (mechanized analysis)And assurance cases
- And suggest ways each might enrich the other

## Three Topics

- Correctness vs. . . . what?
  - Reliability vs. assurance effort
- Logic and epistemology
- Reasoning and communication

## Correctness vs. . . . What?

- Formal verification traditionally tackles a rather narrow goal
- Namely, **formal correctness**
  - One system description satisfies another
  - e.g., an “**implementation**” satisfies its **formal specification**
- Many important issues are **outside this scope**
  - Is the specification relevant, correct, complete, traceable to higher goals, is it formalized correctly?
  - Does the “implementation” (e.g., defined over formal semantics for C) correspond to the actual behavior of the system, with its compiler, operating system, libraries, hardware, etc?
  - And do any assumptions and associated formalized models correctly and adequately characterize the environment?
- But these **are** included in an assurance case
- So what is the **property established by an assurance case**?

## Call it Perfection

- An assurance case establishes certain **critical claims**
  - Often about safety, sometimes security or other concerns
- We want no (or few?) critical **failures**
- Failures concern executions, they're a **dynamic property**
- We're after a **static property** of the design and implementation of the system
- Failures are caused by **faults**, so the property we want is freedom from (critical) faults
- Call that **perfection**
- **A perfect system will never experience a critical failure in operation, no matter how much operational exposure it has**

## Correct but Imperfect Software: Example

- Fuel emergency on Airbus A340-642, G-VATL, on 8 February 2005 (AAIB SPECIAL Bulletin S1/2005)
- Toward the end of a flight from Hong Kong to London: two engines flamed out, crew found certain tanks were critically low on fuel, declared an emergency, landed at Amsterdam
- Two Fuel Control Monitoring Computers (FCMCs) on this type of airplane; each a self-checking pair with a backup (so 6-fold redundant in total); they cross-compare and the “healthiest” one drives the outputs to the data bus
- Both FCMCs had fault indications, and one of them was unable to drive the data bus
- Unfortunately, this one was judged the healthiest and was given control of the bus even though it could not exercise it
- The backups were suppressed because the FCMCs indicated they were not both failed

## Reliability vs. Assurance Effort

- The world is uncertain, so top level claim is often stated quantitatively
  - E.g., no catastrophic failure in the lifetime of all airplanes of one type (“in the life of the fleet”)
  - Or no release of radioactivity in 10,000 years of operation
- And these lead to systems-level requirements for subsystems stated in terms of reliabilities or probabilities
  - E.g., probability of failure in flight control  $< 10^{-9}$  per hour
  - Or probability of failure on demand for reactor protection less than  $10^{-6}$
- For the more demanding probabilities, we do more assurance, or more intensive assurance (i.e., more assurance effort)
- A conundrum: what is
  - The relationship between assurance effort and reliability?



## The Conundrum Illustrated: The Example of Aircraft

- Aircraft **failure conditions** are classified in terms of the severity of their consequences
- **Catastrophic** failure conditions are those that could prevent continued safe flight and landing
- And so on through **severe major, major, minor**, to **no effect**
- Severity and probability/frequency must be **inversely related**
- AC 25.1309: **No catastrophic failure conditions in the operational life of all aircraft of one type**
- Arithmetic and regulation require the probability of catastrophic failure conditions to be less than  **$10^{-9}$  per hour**, sustained for many hours
- And  $10^{-7}$ ,  $10^{-5}$ ,  $10^{-3}$  for the lesser failure conditions

## The Conundrum Illustrated: Example of Aircraft (ctd.)

- DO-178BC identifies five **Software Levels**
- And **71** assurance **objectives**
  - E.g., documentation of requirements, analysis, traceability from requirements to code, test coverage, etc.
- More objectives (plus **independence**) at higher levels
  - **26** objectives at DO178C **Level D** ( $10^{-3}$ )
  - **62** objectives at DO178C **Level C** ( $10^{-5}$ )
  - **69** objectives at DO178C **Level B** ( $10^{-7}$ )
  - **71** objectives at DO178C **Level A** ( $10^{-9}$ )
- The Conundrum: how does doing **more** correctness-based objectives relate to **lower** probability of failure?

## **Some Background and Terminology**

## Aleatory and Epistemic Uncertainty

- Aleatory or irreducible uncertainty
  - is “uncertainty in the world”
  - e.g., if I have a coin with  $P(heads) = p_h$ , I cannot predict exactly how many heads will occur in 100 trials because of randomness in the world

Frequentist interpretation of probability needed here

- Epistemic or reducible uncertainty
  - is “uncertainty about the world”
  - e.g., if I give you the coin, you will not know  $p_h$ ; you can estimate it, and can try to improve your estimate by doing experiments, learning something about its manufacture, the historical record of similar coins etc.

Frequentist and subjective interpretations OK here

## Aleatory and Epistemic Uncertainty in Models

- In much scientific modeling, the **aleatory** uncertainty is captured conditionally in a **model with parameters**
- And the **epistemic** uncertainty centers upon the **values of these parameters**
- As in the coin tossing example:  $p_h$  is the parameter

## Software Reliability

- Not just software, any artifacts of comparably **complex design**
- Software contributes to system failures through faults in its requirements, design, implementation—**bugs**
- A bug that leads to failure is **certain** to do so whenever it is encountered in similar circumstances
  - **There's nothing probabilistic about it**
- Aaah, but the **circumstances** of the system are a **stochastic process**
- So there is a **probability** of encountering the circumstances that activate the bug
- Hence, probabilistic statements about software reliability or failure are perfectly reasonable
- Typically speak of probability of **failure on demand** (pfd), or **failure rate** (per hour, say)

## Testing and Software Reliability

- The basic way to determine the reliability of given software is by experiment
  - Statistically valid random testing
  - Tests must reproduce the operational profile
  - Requires a lot of tests
- Feasible to get to *pdf* around  $10^{-3}$ , but not much further
  - $10^{-9}$  would require 114,000 years on test
- Note that the testing in DO-178C is not of this kind
  - it's coverage-based unit testing: a local correctness check
- So how can we estimate reliability for software?

**Back To The Main Thread**



## Assurance is About Confidence

- We do perfection-based software assurance
- And do more of it when higher reliability is required
- But the amount of perfection-based software assurance has no obvious relation to reliability
- And it certainly doesn't make the software “more perfect”
- Aha! What it does is make us more confident in its perfection
- And we can measure that as a subjective probability

## Possibly Perfect Software

- You might not believe a given piece of software **is** perfect
- But you might concede it has a **possibility** of being perfect
- And the **more assurance** it has had, the **greater that possibility**
- So we can speak of a (subjective) **probability** of perfection
- For a frequentist interpretation: think of all the software that **might** have been developed by comparable engineering processes to solve the same design problem
  - **And that has had the same degree of assurance**
  - **The probability of perfection is then the probability that any software randomly selected from this class is perfect**

## Probabilities of Perfection and Failure

- Probability of perfection relates to **software assurance**
- But it also relates to **reliability**:

By the formula for total probability

$$\begin{aligned} P(\text{s/w fails [on a randomly selected demand]}) & \quad (1) \\ &= P(\text{s/w fails | s/w perfect}) \times P(\text{s/w perfect}) \\ & \quad + P(\text{s/w fails | s/w imperfect}) \times P(\text{s/w imperfect}). \end{aligned}$$

- The **first term** in this sum is zero, because the software does not fail if it is perfect (**other properties won't do**)
- Hence, define
  - $p_{np}$  probability the software is imperfect
  - $p_{fnp}$  probability that it fails, if it is imperfect
- Then  $P(\text{software fails}) = p_{fnp} \times p_{np}$
- This analysis is **aleatoric**, with parameters  $p_{fnp}$  and  $p_{np}$

## Epistemic Estimation

- To apply this result, we need to assess values for  $p_{fnp}$  and  $p_{np}$
- These are most likely **subjective probabilities**
  - i.e., degrees of belief
- Beliefs about  $p_{fnp}$  and  $p_{np}$  **may not be independent**
- So will be represented by some joint distribution  $F(p_{fnp}, p_{np})$
- Probability of software failure will be given by the Riemann-Stieltjes integral

$$\int_{\substack{0 \leq p_{fnp} \leq 1 \\ 0 \leq p_{np} \leq 1}} p_{fnp} \times p_{np} dF(p_{fnp}, p_{np}). \quad (2)$$

- If beliefs **can** be separated  $F$  factorizes as  $F(p_{fnp}) \times F(p_{np})$
- And (2) becomes  $P_{fnp} \times P_{np}$

Where these are the **means of the posterior distributions** representing the assessor's beliefs about the two parameters

## Practical Application—Nuclear

- Traditionally, nuclear protection systems are assured by statistically valid random testing
- Very expensive to get to  $pdf$  of  $10^{-4}$  this way
- Our analysis says  $pdf \leq P_{fnp} \times P_{np}$
- They are essentially setting  $P_{np}$  to 1 and doing the work to assess  $P_{fnp} < 10^{-4}$ 
  - Conservative assumption that allows separation of beliefs
- Any software assurance process that could give them  $P_{np} < 1$ 

Would reduce the amount of testing they need to do

  - e.g.,  $P_{np} < 10^{-1}$ , which seems very plausible
  - Would deliver the the same pdf with  $P_{fnp} < 10^{-3}$
- This could reduce the total cost of certification
  - **Conservative methods available** if beliefs not independent

## Practical Application—Aircraft, Version 1

- Aircraft software is assured by processes such as DO-178C Level A, needs failure rate  $< 10^{-9}$  per hour
- They also do a massive amount of all-up testing but do not take assurance credit for this
- Our analysis says software failure rate  $\leq P_{fnp} \times P_{np}$
- So they are setting  $P_{fnp} = 1$  and  $P_{np} < 10^{-9}$
- No plane crashes due to software, enough operational exposure to validate software failure rate  $< 10^{-7}$ , even  $10^{-8}$
- Does this mean flight software has probabilities of imperfection  $< 10^{-7}$  or  $10^{-8}$ ?
- And that DO178C delivers this?

## Practical Application—Aircraft, Version 2

- That seems unlikely; an alternative measure is  $p_{srv}(n)$ , the probability of surviving  $n$  demands without failure, where

$$p_{srv}(n) = (1 - p_{np}) + p_{np} \times (1 - p_{fnp})^n \quad (3)$$

i.e., probability of failure-free operation over long periods remains constant with high probability of perfection, but decays exponentially for imperfect but reliable

- Cannot do  $10^{-9}$  this way
- But can make  $n$  equal to “life of the fleet” and get there with modest  $p_{np}$  and  $p_{fnp}$
- Need a “bootstrap” for  $p_{fnp}$  to have confidence in first few months of flight, and could get that from the all-up system and flight tests
- Thereafter, experience to date provides confidence for next increment: see paper by Strigini and Povyakalo

## Practical Application: Two Channel Systems

- Many safety-critical systems have two (or more) diverse “channels” arranged in 1-out-of-2 (1oo2) structure
  - E.g., nuclear shutdown
- A primary protection system is responsible for plant safety
- A simpler secondary channel provides a **backup**
- **Cannot** simply multiply the pfd's of the two channels to get pfd for the system
  - Failures are **unlikely** to be **independent**
  - E.g., failure of one channel suggests this is a difficult case, so failure of the other is more likely
  - Infeasible to measure amount of dependence

So, traditionally, difficult to assess the reliability delivered



## Two Channel Systems and Possible Perfection

- But if the second channel is simple enough to support a plausible claim of possible perfection, then
  - Its imperfection is conditionally independent of failures in the first channel at the aleatory level
  - Hence, system pfd is conservatively bounded by product of pfd of first channel and probability of imperfection of the second
  - $P(\text{system fails on randomly selected demand}) \leq pfd_A \times pnp_B$

This is a theorem

- Epistemic assessment similar to previous case
  - But may be more difficult to separate beliefs
  - Conservative approximations are available

## Type 1 and Type 2 Failures in 1oo2 Systems

- So far, considered only failures of **omission**
  - **Type 1** failure: both channels fail to respond to a demand
- Must also consider failures of **commission**
  - **Type 2** failure: either channel responds to a **non**demand
- Demands are events at a **point** in time; nondemands are absence of demands over an **interval** of time
- So full model must unify these
- Details straightforward but lengthy

## Monitored Architectures

- A variant on 1oo2
- One **operational** channel does the business
- Simpler **monitor** channel can shut it down if things look bad
- Used in airplanes, avoids **malfunction** and **unintended function**
  - Higher level redundancy copes with **loss of function**
- Analysis is a variant of 1oo2:
  - No Type 2 failures for operational channel
- Monitored architecture **risk** per unit time
$$\leq c_1 \times (M_1 + F_A \times P_{B1}) + c_2 \times (M_2 + F_{B2|np} \times P_{B2})$$
where the  $M$ s are due to mechanism shared between channels
- May provide justification for some of the architectures suggested in ARP 4754
  - e.g.,  $10^{-9}$  system made of Level C operational channel and Level A monitor

## Monitors Do Fail

- Fuel emergency on [Airbus A340-642](#), G-VATL, 8 February 2005 (already discussed)
  - Type 1 failure
- EFIS Reboot during spin recovery on [Airbus A300](#) (American Airlines Flight 903), 12 May 1997
  - Type 2 failure
- These weren't very good monitors
- So what's to be done? ... hold that question

## Diagnosis and Prescriptions

- Need a framework for discussing whole process of assurance
- Idea of an **assurance case** provides this
  - Claims
  - Argument
  - Evidence
  - The **argument** justifies the **claims**, based on the **evidence**
- Some fields require assurance or safety case for certification
  - e.g., FDA requires them for **Infusion pumps**
- Others use **standards** and **guidelines** such as DO-178C
  - The **claims** are largely established by regulation, guidelines specify the **evidence** to be produced, and the **argument** was presumably hashed out in the committee meetings that produced the guidelines
  - In the **absence of a documented argument**, it's not clear what some of the evidence is for: e.g., MC/DC testing

## Assurance Cases and Formal Verification

- The **argument** justifies the **claims**, based on the **evidence**
- This is a bit like **logic** (cf. “argumentation” later)
  - A **proof** justifies a **conclusion**, based on given **assumptions and axioms**
- So what’s the (next) **difference** between an assurance case and a formal verification?
- Aha! An assurance case also closely examines the **interpretation** of the **formalized assumptions** and **conclusion** and why we should **believe** the **assumptions and axioms**
  - e.g., contemplate my formal verif’n in PVS of Anselm’s **Ontological Argument** (for the existence of God)
- We could **expand** formal verification to include the elements traditionally outside its scope, and attention would then focus on **credibility of their representation in logic**

## Logic And The Real World

- Formal verification is **calculation in logic**
  - It's difficult because calculations in logic are all NP-Hard
  - But benefits are the same as those for calculation in other engineering fields (can **consider all cases**)
- Software **is** logic
- But it interacts with the **world**
  - What it is supposed to do (i.e., **requirements**)
  - The **actual semantics** of its implementation
  - **Uncertainties** and **hazards** posed by sensors, actuators, devices, the environment, people, other systems

We must consider what we **know** about all these, and how we represent them

- For formal verification we describe them by **models**, in logic

## Logic and Epistemology in Assurance Cases

- We have just **two sources of doubt** in an assurance case
- **Logic doubt**: the validity of the argument
  - Can be **eliminated** by formal verification
  - Subject to caveats on soundness of methods & tools
  - This is **Leibniz' Dream**: “let us calculate”
- **Epistemic doubt**: the accuracy and completeness of our knowledge of the world in its interaction with the system
  - As expressed in our models and requirements
  - **This is where we need to focus**
- Same distinction underlies **Verification** and **Validation** (V&V)
  - Did I build the system right?
    - ★ **Did I truly prove the theorems?**
  - Did I build the right system?
    - ★ **Did I prove the right theorems?**



## Aside: Resilience

- It is often possible to **trade** epistemic and logic doubts
  - Weaker assumptions, **fewer** epistemic doubts
  - But more complex implementations, **more** logic doubt
- For example, **highly specific** fault assumptions, vs. **Byzantine fault tolerance**
- I claim **resilience** is about **favoring weaker assumptions**
- Good for **security** also: the bad guys **attack your assumptions**
- Formal verification lets us cope with the added logic doubt
  - cf. FAA disallows adaptive control due to logic doubt

## Reducing Epistemic Doubt: Validity

- We have a model and we want to know if it is **valid**
- One way is to run experiments against it
- That's why **simulation models** are popular
  - To be executable, have to include a lot of detail
  - But detail is not necessarily a good thing in a model
  - **Epistemic doubt** whether real world matches all that detail
- Instead we should favor descriptions in terms of **constraints**
  - Our task is to **describe** the world, **not to implement it**
  - Less is more!
- **Calculation on constraint-based models is now feasible**
  - Recent advances in fully automated verification
  - **Infinite bounded model checking** (Inf-BMC), enabled by solvers for **satisfiability modulo theories** (SMT)
- Cf. **equivalence checking** on (coercive) **reference implementations**, vs. **constraint checking** on **loose models**

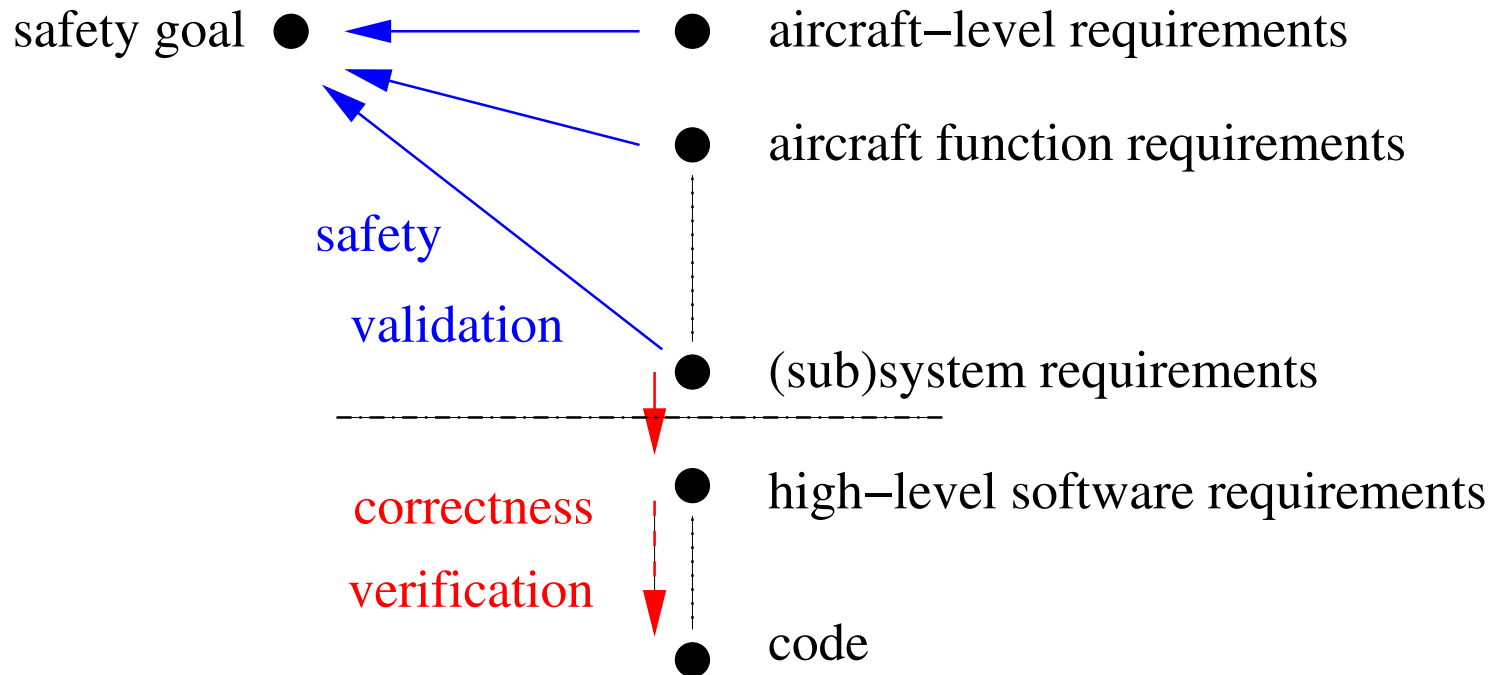
## Reducing Epistemic Doubt: Validity (ctd. 1)

- **All** aircraft incidents due to software had their root cause in **flawed requirements**
  - Either the **system level requirements** were wrong
  - Or the **high level software requirements** did not correctly reproduce their intent
- **None** were due to implementation defects
  - Might not be so in other application areas
- One problem is that descriptions at the system level are (rightly) very abstract
  - **Typically box and arrow pictures, supplemented with math**
  - Little support for automated exploration and analysis
- And these descriptions are getting more complex, because there are more cases to deal with (i.e., **more like software**)

## Reducing Epistemic Doubt: Validity (ctd. 2)

- Traditional ways to explore system-level models, such as **failure modes and effects analysis** (FMEA) and **fault tree analysis** (FTA) can be seen as **manual** ways to do incomplete state exploration with some heuristic focus that directs attention to the paths most likely to be informative
- Modern system models have increasingly many cases, like software. so it makes sense to **apply methods from software** to the specification and analysis of these designs
- But must keep things abstract
- **Aha!** Inf-BMC can do this
- Inf-BMC allows use of **uninterpreted functions**, e.g.,  $f(x)$
- Constraints can be encoded as **synchronous observers**
- With comparable models Inf-BMC can do **automated** model checking and cover the **entire** modeled space

# Traditional Division of System and Software Assurance



- As more of the system design goes into software
- Software analysis methods should be applied to system req'ts

## Reducing Epistemic Doubt: Completeness

- Quintessential completeness problem is hazard analysis
  - Have I thought of **all** the things that could go wrong?
- There are systematic techniques that help suggest possible hazards: FMEA, HAZOP etc.
  - These can be partially automated
  - cf. notion in Epistemology that **knowledge** is **belief justified** by a **generally reliable method**
- But there seems no way to **prove** we do have all the hazards
- So surely need some **measure** of our confidence that we do
- Same for all the other reasons (called **defeaters**) why our safety argument might be flawed

## Eliminative Induction, Baconian Probability

- Some take inspiration from [scientific method](#)
- Many candidate theories, design experiments to test them, [eliminate](#) those shown to be wrong (Francis Bacon, roughly)
- “Once you eliminate the impossible, whatever remains, no matter how improbable, must be the truth” (Holmes)
- Substitute defeaters for theories
  - [Have many reasons why safety argument could be flawed, eliminate them one by one](#)
- [Baconian Probability](#) is a measure for this:  
[number eliminated ÷ number considered](#)
- More complex form advocated in Philosophy of Law (Cohen)
  - “Beyond reasonable doubt,” “balance of probabilities”
- [Doesn't behave like a probability](#)

## Bayesian Induction

- An intellectually justifiable method should allow us to quantify
  - Confidence that we have identified **all** defeaters
  - Confidence that we have eliminated or mitigated **any given** defeater
  - A way to **apportion effort**: confidence required in the elimination of any given defeater should depend on the risk (i.e., likelihood and consequence) that it poses
- Surely the right way to do this is to use **genuine probabilities**
  - Subjective prior probabilities updated (via Bayes rule) as evidence becomes available
- “**Bayesian Induction is Eliminative Induction**” (Hawthorne)
- Making this practical would be a significant research agenda



## Reasoning and Communication

- I've focused on the idea that an assurance case is about **reasoning**: it should be a deductively sound argument
- But an assurance case is not (just) a proof
- It also has to unite human stakeholders in **shared understanding and belief**
- And there's a separate tradition called **argumentation** that focuses on these **communication** aspects within logic
- e.g., Toulmin-style argumentation, Dung-style argument structures, defeasible reasoning, etc.
- My belief is that communication is best assisted by **active exploration** (e.g., "**what-if**") and this is supported by automated support for the deductive aspect
  - Toulmin had same technology as Aristotle: a printed page
- But there's excellent scope for exploration and research here

## Conclusion

- **Probability of perfection** is a radical and valuable idea
  - It's due to Bev Littlewood, and Lorenzo Strigini
- Provides the bridge between correctness-based verification activities and probabilistic claims needed at the system level
- Explains what software assurance **is**
- Relieves formal verification, and its tools, of the **burden of infallibility**
- Explains the merit of **monitors**
- Distinguishing **logic and epistemic doubts** allows different methods to be focused on each
- Possibly explains **resilience**
- Suggests approaches for **reducing epistemic doubts**
- And for **quantifying confidence** in total case

## Proposals: Practical and Speculative

- Use monitors **formally verified or synthesized** against the **system-level** safety requirements
- Use **formal methods** in analysis of **system-level** designs and requirements
- Develop **a priori** estimates of probability of perfection based on assurance performed
  - May be able to compose estimates from each element of the case (e.g., each objective of DO-178C), BBN-style
- **Combine** testing and correctness-based software assurance in estimating reliability
- Develop an **intellectually justifiable** approach to certification
- But note that **none** of this is **compositional**: fix that!
- Unify, or at least harmonize, the **reasoning** and **communication** aspects of assurance cases