Synthesis Using EF-SMT Solvers

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EF-SMT or $\exists\forall$-SMT

$$\exists \vec{a} : \forall \vec{x} : \phi(\vec{a}, \vec{x})$$

Important factor: theory of ϕ

Many synthesis problems can be transformed into $\exists\forall\phi$ problems

Sat/SMT is to verification as QBF/EF-SMT is to synthesis

Synthesis

Verification: Given M, ϕ, determine if $M \models \phi$

Synthesis: Given partial model $M(?)$ and ϕ, determine if there exists a completion M of $M(?)$ s.t. $M \models \phi$

Value: Applications where completing $M(?)$ is not intuitive/easy

Philosophy: Lift formal verification techniques to synthesis

Synthesis as $\exists\forall\phi$: Version 1

Take inspiration from bounded model checking:

$$M(\vec{c}, \vec{u}) \models \phi$$
$$\Uparrow$$
$$M(\vec{c}, \vec{u}) \models \phi^k, \qquad \text{for some } k$$
$$\Uparrow$$
$$\exists \vec{c} : \forall \vec{x_0}, \vec{x_1}, \ldots, \vec{x_k} : \exists \vec{u_0}, \ldots, \vec{u_k} : \text{ quantifier-free FO formula}$$
$$\Uparrow$$
$$\exists \vec{a}, \vec{c} : \forall \vec{x} : \exists \vec{u} : \text{ quantifier-free FO formula}$$

Synthesis as $\exists\forall\phi$: Version 2

Take inspiration from induction- and abstraction-based verification:

$$M(\vec{c},\vec{u}) \models \phi$$
$$\Uparrow$$
$$\exists\Phi,\vec{c},\vec{u} : ((M(\vec{c},\vec{u}) \models \Phi) \ \wedge \ (\Phi \Rightarrow \phi))$$
$$\Uparrow$$
$$\exists\Phi,\vec{c} : \forall\vec{x} : \exists\vec{u} : \ \text{quantifier-free FO formula}$$
$$\Uparrow$$
$$\exists\vec{a},\vec{c} : \forall\vec{x} : \exists\vec{u} : \ \text{quantifier-free FO formula}$$

The last step is performed by choosing a template for Φ

Verification :: Synthesis

Verification : $\exists\forall$
does there exist a certificate that proves the property for the system?

Synthesis: $\exists\forall\exists$
synthesis variables that are independent of the state go inside the first $\exists$
synthesis variables that are dependent on the state go inside the second $\exists$

Usually link verification to one and synthesis to two quantifiers

Note: Not just parameter synthesis, the synthesis variables could encode how subsystems are composed...

Synthesis Challenge

Automated synthesis is difficult

- Search space for models is very large
  domain of first $\exists$ quantifier large
- For each potential model, we have to solve a verification problem
  inner $\forall$ problem difficult

Overcoming the Synthesis Challenge

- Reduce outer $\exists$ complexity:
  - Use templates to reduce synthesis search space
  - Bound all synthesis variables
- Reduce inner $\forall$ complexity:
  - consider domains where verification is easy (for e.g., straight-line programs)
  - Replace "verification" check by a weaker check (as in BMC)
  - Search for small (strong) verification proofs

Syn1: Straight-line Program Synthesis

Problem: Given library functions f_i's, and a desired f, find a way to compose f_i's to generate f.

$$\exists P : \forall x : f(x) = P(x), \quad P \text{ a straight-line program composing } f_i\text{'s}$$
$$\Downarrow$$
$$\exists \pi : \forall x : F_{\text{spec}}(x) = f_{\pi(1)}(f_{\pi(2)}(f_{\pi(3)}(x)))$$

Reduce synthesis search space:

- fix length of program
- fix upper bound on number of each f_i

Verification problem: Straight-line program equivalence

Syn1 Example

Specification: Turn-off rightmost contiguous 1 bits

Example: 010101100 $\mapsto$ 010100000

Budget: two addition, at most four bitwise Boolean operators, constant 1, -1

Synthesized Program:

1. $o_1 := x + (-1)$;
2. $o_2 := o_1 | x$;
3. $o_3 := o_2 + 1$;
4. return $o_3 \& x$;

Correctness on sample input:

$$010101100 \mapsto 010101011 \mapsto 010101111 \mapsto 010110000 \mapsto 010100000$$

Other examples: Bitvector tricks from Hacker's delight

Synthesis: $\exists\forall$ bit-vector theory

Syn1.1: Loop-free Program Synthesis

Problem: Given library functions f_i's including an if-then-else, and a desired f, find a way to compose f_i's to generate f.

Step to feasibility: As in Syn1.

Syn1.1 Example

Specification: Obfuscated code

Example: We are given

```
if (h(x))
   if (x*(x+1)% 2 == 1) y := f(x) else y := g(x)
else y := f(g(x))
```

Components Budget: f, g, h, if-then-else

Synthesized Program:

```
o := g(x);
if (h(x)) y := o; else y := f(o);
```

Correctness: Equivalence of the two loop-free programs

Syn2: Loop Synthesis

Problem: Given incomplete loop body and desired post, find a way to complete the loop body so that correctness holds.

procedure product(x_0, y_0):
    $s := 0$;
    $y := y_0$;
    while $(y \neq 0)$:
        $s := c_1 x_0 + c_2 y_0 + c_3 y + c_4 s + c_5$;
        $y := y - 1$;
    return s; $[s = x_0 * y_0]$

Syn2: Synthesize Code and Correctness

Let loop invariant be an arbitrary degree 2 polynomial

$$a_0 x_0^2 + a_1 y_0^2 + a_2 y^2 + a_3 s^2 + a_4 x_0 y_0 + \ldots + a_{14}$$

Now the synthesis formula is:

$\exists c_1, \ldots, c_5 :$
$\exists a_0, \ldots, a_1 4 :$
$\forall x_0, y_0, s, y :$
    loop-invariant is indeed a loop invariant
    loop-invariant and negation of condition implies post

$\exists\forall$ theory of reals

Syn3: Fault-tolerant Distributed Algorithms

A set of m (communicating) machines arranged in some topology want to satisfy a given temporal property. Given the state space of the machines, find the state update function for the m machines.

Do so assuming some f are faulty

Fault assumptions can be malicious, asymmetric, transient.

Both the synthesis search space, and the verification problem need to be simplified

Syn3 Example

Each machine has a private Boolean value v_i

Each machine has a vector of Boolean values – its knowledge about the private value of all the other machines – consistency vector

Interactive consistency: n processes, out of which at most f can be faulty, satisfy interactive consistency if the consistency vectors of some $n - f$ processes are identical

Desired property: FG(interactive consistency)

Each machiine updates its v_i by $f_i(v'_1, \ldots, v'_m)$, where v'_j is the value it receives from j-th machine (may not equal v_j if j is faulty)

$$\exists f_i : S_1 || \ldots || S_m \models \mathrm{FG}(\text{interactive-consistency})$$

Syn3 Example: Simplifying Verification

Typically also want algorithms to be self-stabilizing

I.e., initial state can be arbitrary

In this case, $\mathsf{FG}\phi$ is implied by, say, $\mathsf{XXXX}\phi$

Thus, for a fixed m, we get an 2-QBF formula

We can synthesize Pease, Shostak, and Lamport's algorithm for interactive consistency that works in presence of permanent, malicious and asymmetric (Byzantine) faults

Other examples: Dijkstra's self-stabilizing mutual exclusion algorithm, interactive consistency for transient Byzantine faults

Syn4: Circuit Understanding

Goal: Discover high-level description of a given combinational circuit (verilog netlist)

Permutation Independent Equivalence Checking: Is circuit equivalent to, say an adder, for some permutation of the input and output signals

Permutation Independent Conditional Equivalence Checking: Is circuit equivalent to, say an adder, for some permutation of the input and output signals and some setting of the control input signals

Syn4: Simplifications

Real challenge for large netlists

Reducing the synthesis search space

- templates – user can specify subsets of input signals whose permutation may be one input word; same for control and output signals
- signatures – necessary conditions on the synthesis variables obtained from analyzing the target function
  some over-approximation of QE of $\forall\phi$
  type constraints that eliminate solutions (that will not work)

Syn4: Examples

Flattened netlists generated using Synopsys Design Compiler

From high-level behavioral verilog collected from ISCAS'85 benchmarks, ALU from a academic processor implementation, etc.

Template library contains adders, subtracters, shifters, multipliers, counters of varying bitwidths

Equal number of positive and negative synthesis instances used

Successfully solved 37/40 instances with QBF preprocessor Bloqqer and EF-Yices

Syn5: Security Schemes

Goal: Automatically synthesize encryption schemes, message exchange protocols, etc.

Similar to straight-line program synthesis

Typical protocols are 10-20 line programs

Requirement: Meet some security level

A Template Language

A class of straight-line programs with $na + nb + 4$ lines:

```
(library
  (G 1) (H 1) (oplusr 2) (oplus 2) (identity 1))
(blocks
   (lm 1 ((input m::(bool-to-bv false false false false true))))
   (lr 1 ((input r::(bool-to-bv false false false true false))))
   (l1 na ( (oplusr (lm -) (lr -)) (G (lr -)) (H (lm -))))
   (l2 2 ( (identity (l1) ) ))
   (l3 nb ( (oplus (l2 -) (l2)) (H (l2)) (G (-)) )))
```

Template Language Features

- broad in its range: from writing concrete program to arbitrary program of length l
- hence, can restrict the synthesis search space
- length of program (blocks) can be left parametric
- Each variable is associated to two entities – its value and its type
- Can include type restrictions to further restrict synthesis search space, and also describe properties
- Is internally compiled into EF-Yices: can use arbitrary yices expressions too

Syn5: Example

Synthesizing OEAP encryption scheme

Using the part template shown above

Interpret messages over bitvectors, and all operations mapped to bitvector operations

Define a type that conservatively tracks which terms are essentially random

Requirement: Output terms are 'random' and there is an efficient decryption algorithm

Syn5: Examples of Synthesized Schemes

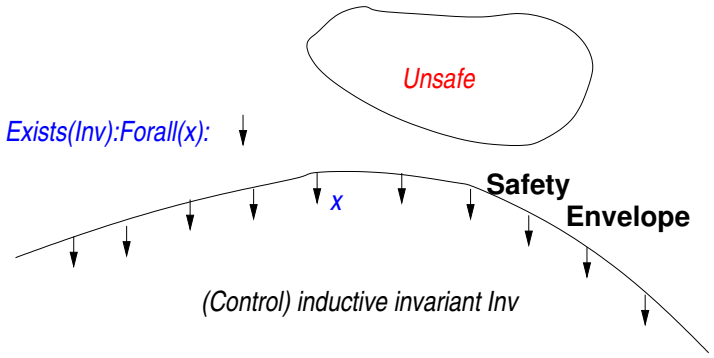
- $f(G(r)||m + G(r))$
- $f(r||m + G(r))$
- $f(r + H(m)||m + G(r + H(m)))$
- $f(G(r + H(m))||r + G(r + H(m)))$
- $f(m + G(r)||H(m + G(r)) + r)$

Why?

- padding with zero not modeled
- and the resulting security issues

Syn6: Controller Synthesis for Safety

Φ = (Global) Control inductive invariant



$\exists$(control inductive invariant that implies safety)

$\exists(set)$: (set is a control inductive invariant) and (set implies safety)

$\exists(V(\vec{x}) \geq 0) : \forall \vec{x} : \exists u : (V(\vec{x}) = 0 \Rightarrow V(F(\vec{x}, u, 0^+)) \geq 0)$

$\wedge(V(\vec{x}) \geq 0 \Rightarrow \text{Safe}(\vec{x}))$

Syn6: Controller Synthesis for Safety

Two things:

- Safe controller will be a fallback controller, so as well make it "bang-bang":

$$\exists \vec{a}, \vec{c} : \forall \vec{x} : \exists \vec{u} : \phi$$
$$\Uparrow$$
$$\exists \vec{a}, \vec{c} : \forall \vec{x} : (\phi(u_{min}) \vee \phi(u_{max}))$$

- Rewrite $V(F(\vec{x}, u, 0^+)) \geq 0$ to eliminate F

Syn6: Controller Synthesis for X

Can be similarly converted to an $\exists\forall\exists$ and if domain of u is finite, then to an $\exists\forall$ problem

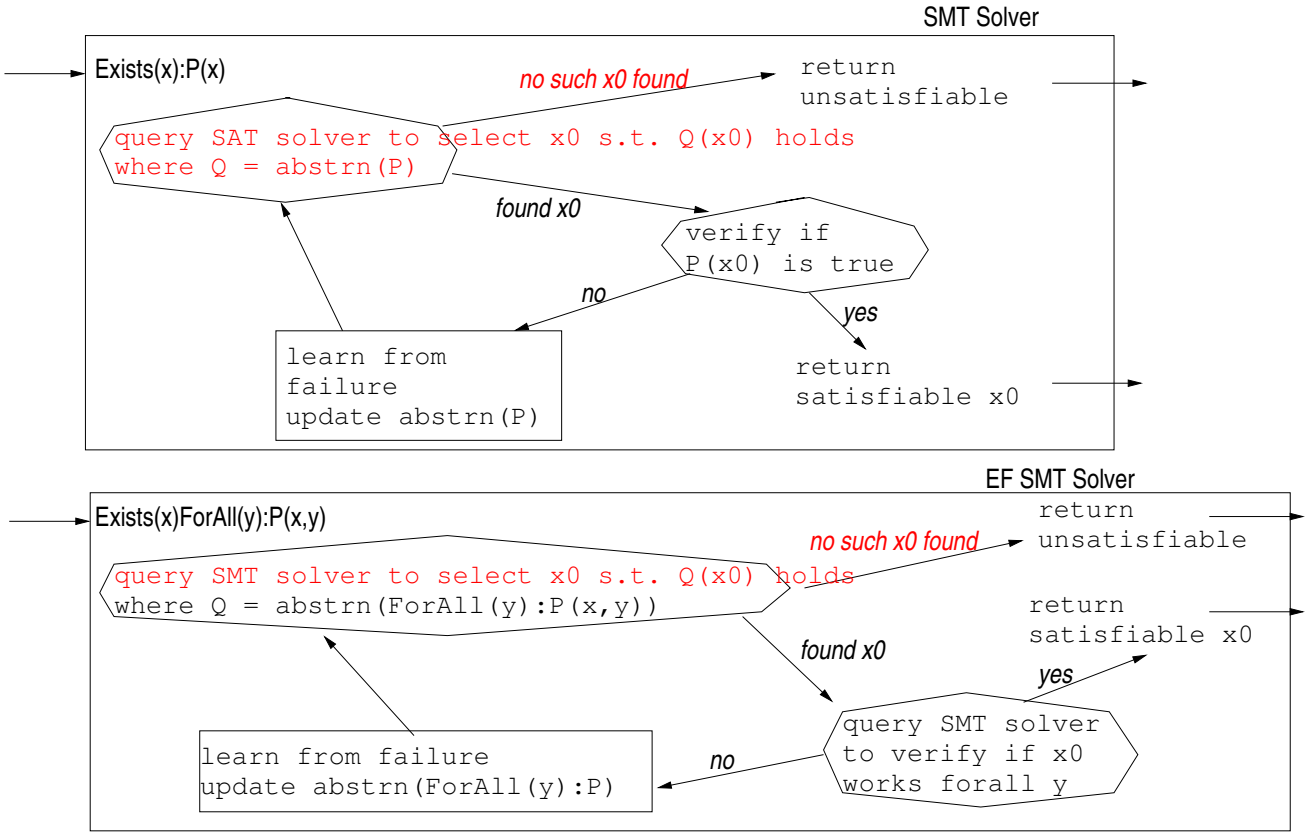
Applications:

- switching logic synthesis
- adaptive cruise controller

How to Solve $\exists\forall$ Problems?

EF Solving

Constraint Solving Using Abstraction Refinement
Guided by Learning via Counter Examples



EF Solving: Learning

$$\exists x : E(x) \wedge \forall y : B(x, y) \Rightarrow C(x, y)$$

Find a s.t. $E(a)$; if none, then return Unsat

Find b s.t. $B(a, b) \wedge \neg C(a, b)$; if none, return Sat

Update $E := E \cup \{B(x, b) \Rightarrow C(x, b)\}$

Convergence can be slow

perform approximate quantifier elimination on $\forall y : B(x, y) \Rightarrow C(x, y)$ to get constraints on x

E.g., equality resolution

Terminates for finite domains

EF Solving: Arithmetic Domains

Several different approaches:

- dedicated QE procedures
- Use duality theorems to replace $\forall$ by $\exists$
- Some combination of simplification, QE, simulation,

EF Solving: Simulation Guided Synthesis

$$\exists \vec{a} : \forall \vec{x} : (V \geq 0 \Rightarrow gap \geq 0) \ \wedge \ (V = 0 \Rightarrow dV/dt > 0)$$

1. $S :=$ Sample the reachable state space
   - simulation/trace data
   - constraint solving on guard for $\forall$
2. Solve $\exists \vec{a} : \bigwedge_{\vec{x} \in S} : V \geq 0$
   - Find a suitably large safety envelope that contains all points
   - Using the SMT solver Yices – note linear
3. Shrink the safety envelope minimalistically to guarantee safety
   - All but one $\exists$ variables is fixed
   - Constant term d is the only $\exists$ variable remaining

- Use quantifier elimination

EF Solving: NRA Fragment

ϕ: Nonlinear real arithmetic

$$\exists \vec{a} : \forall \vec{x} : \phi(\vec{a}, \vec{x})$$
$$\Uparrow$$
$$\exists \vec{a}, \vec{p} : \forall \vec{x} : \bigwedge_i q_i(\vec{p}, \vec{a}, \vec{x}) = 0$$
$$\Uparrow$$
$$\exists \vec{a}, \vec{b} : \forall \vec{x} : \bigwedge_i r_i(\vec{a}, \vec{b}, \vec{x}) = 0$$
$$\Uparrow$$
$$\exists \vec{a}, \vec{b} : \bigwedge_i s_i(\vec{a}, \vec{b}) = 0$$

So problem reduces to $\exists \vec{a} : \phi(\vec{a})$

EF-NRA $\mapsto$ E-NRA: Example

$$\exists a, b, u, v, w \forall s, x_0, y_0, y, s_1, y_1 :$$

$$s = ax_0y_0 + bx_0y \wedge s_1 = s + x_0 \wedge y_1 = y - 1 \Rightarrow s_1 = ax_0y_0 + bx_0y_1$$

$$\Uparrow$$

$$s_1 - ax_0y_0 - bx_0y_1 = u(s - ax_0y_0 - bx_0y) + v(s_1 - s - x_0) + wx_0(y_1 - y + 1)$$

$$\Uparrow$$

$$1 = v \wedge -a = -ua \wedge -b = w \wedge 0 = u - v \wedge 0 = -ub - w \wedge 0 = -v + w$$

Other Approaches for EF Solving

From "input-output examples":

Finite synthesis:

$$\exists \forall \phi$$
$$\exists \bigwedge_i \phi$$

Distinguishing input technique:

$$\exists \forall \phi$$
$$\exists c \exists c' : \bigwedge_i \phi(c) \wedge \phi(c') \wedge c \neq c'$$

Conclusion

- convert synthesis problem to $\exists\forall$ problem

EF-SMT:

- integrate various different approaches
- into an effective strategy for EF-SMT solving