

# Formal Composition Technology for Time-Triggered Systems

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# Administrative

**Title:** Formal Composition Technology for  
Time-Triggered Systems

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# Subcontractors and Collaborators

Subcontractors: No subcontractors

Collaborations:

- MoBIES:
  - Vanderbilt University: Design and development of interchange format and translators for analysis tools
  - U Penn, CMU: Model exchanges via HSIF
  - Kestrel: Parser for Stateflow
- SEC: Stanford
- BioSpice: SRI
- Outside: Honeywell, NASA

# Problem Description

- Develop tools and techniques for **automated formal analysis** and **assurance** of models of embedded hybrid systems
- Develop **invisible formal methods** technology for integration with modeling tools based on lightweight theorem proving and symbolic reasoning

## Program Objectives

- Provide analysis tools that integrate with the design process for development of embedded systems
- Success criteria: automated analysis on models from the OEP challenge problems

# Tool Description

<b>Name:</b>	SAL tool suite
<b>Description:</b>	Analysis of safety properties of input models
<b>Input:</b>	Model in the SAL language/SAL XML Assumptions: Polynomial hybrid systems
<b>Output:</b>	Abstract system, Other verification results (typechecking), Counter-examples, etc
<b>Interfaces:</b>	Technology: HSIF (VU, UCB, Penn, CMU) Now: Can translate from HSIF <b>into</b> SAL Future: Translate from SAL to HSIF .∴ connects with tools with HSIF interface
<b>Non-MoBIES:</b>	Future planned interface to SBML, etc

# OEP Participation

**OEP:** Berkeley V2V

**Technical POC:** Mike Drew

## Contributions:

- Design of the HSIF interchange format in collaboration with Vanderbilt, Berkeley, U Penn, CMU
- Translator from HSIF into SAL
- Verified simple V2V examples using a novel technique of doing reachability for linear systems

# Project Status: Approach

Our present technical approach to verification of safety properties for hybrid system models is:

- Use invisible formal methods to check for certain kinds of inductive properties for the given model, e.g. type safety, completeness of specification, etc
- Use automated abstraction techniques to get a **discrete** transition system **abstract** model from a hybrid system model
- Output the abstract system (for other tools to use)
- Use the configurable explicit state model-checker to perform analysis on the abstract model
- Translate from and to the SAL modeling language to extend its interface

# Project Status: Progress

- Developed a translator from HSIF to SAL
- Experimenting with the V2V challenge problems, we observed that effective use the abstraction tool for hybrid systems **requires a good set of seed polynomials**
- Developed **new theoretical results** to do non-trivial **reachability** computation on classes of linear systems that do not fall in the class of systems with a decidable reachability problem such as
  - nilpotent systems,
  - diagonalizable with rational eigenvalues,
  - diagonalizable with imaginary eigenvalues

The new result suggests seed polynomials for the abstraction tool

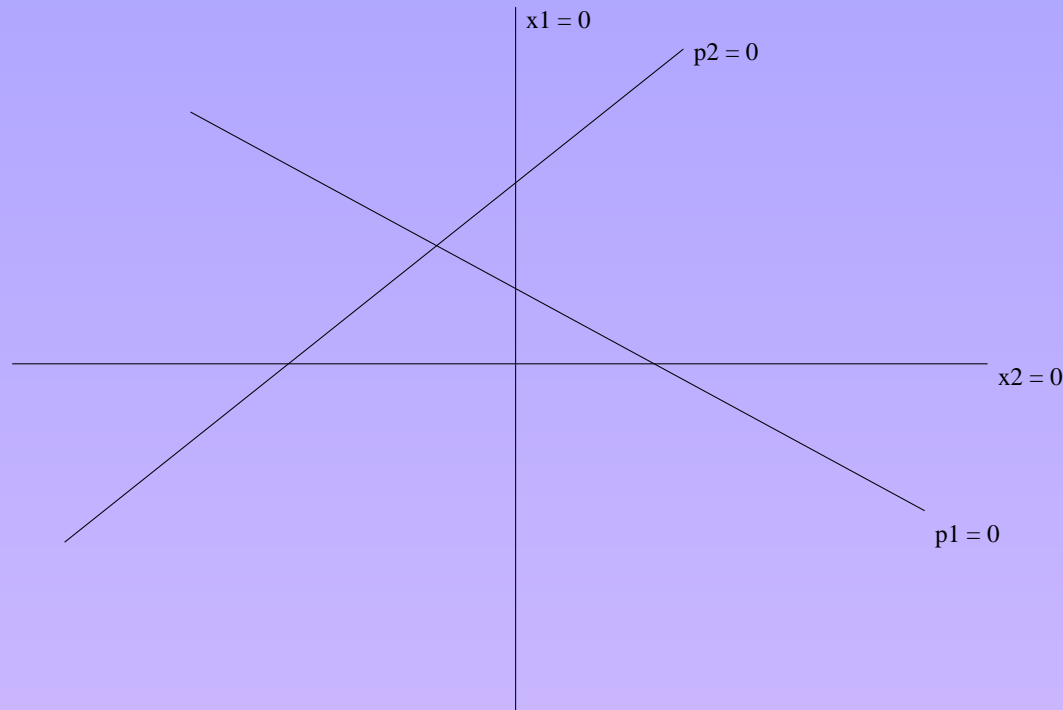


# Project Status: Progress

- Developed a specialized model-checker for dealing with models **created by abstraction**
  - model-checker is aware that certain states might not be **feasible**
  - model-checker interfaces with an external routine to check for feasibility of a particular state
- Implemented the new technique of abstraction and model-checking using a new fast decision procedure for polynomial formulas
- Demonstrated the applicability of the new techniques on collision avoidance examples from the V2V auto OEP

# Project Status: Progress

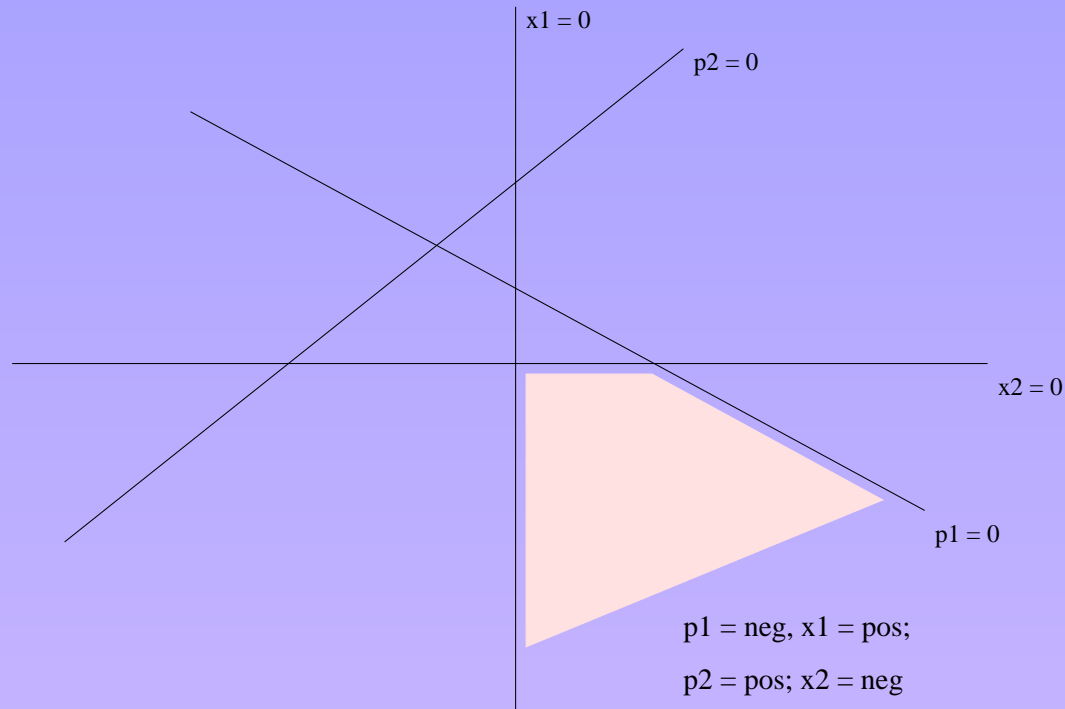
Recall the abstraction technique. A continuous dynamical system with two state variables, with a concrete state space  $\mathbb{R}^2$ :



Partitioned w.r.t signs of polynomials  $x_1$ ,  $x_2$ ,  $p_1$ , and  $p_2$ .

# Abstraction Algorithm

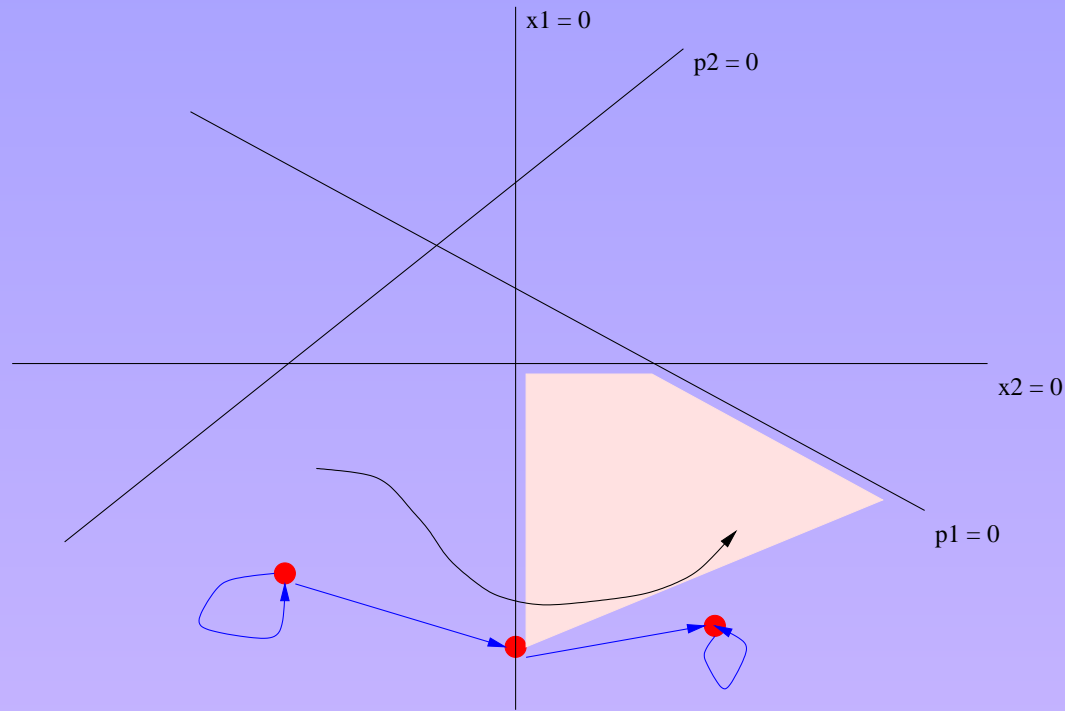
Abstract states correspond to subsets of concrete states.



More polynomials would mean more abstract states.

# Abstraction Algorithm

Abstract transitions overapproximate concrete transitions.



Total number of abstract states =  $3^4 = 81$ , but feasible abstract states =  $11 + 16 + 6 = 33$

# Choosing Partition Polynomials

For a linear system, say specified by matrix  $A$ , use the **eigenvector of the transpose  $A^T$**  corresponding to a **real eigenvalue**.

Example. Consider a cruise control:

$$\begin{bmatrix} \dot{v} \\ \dot{vf} \\ \dot{a} \\ \dot{gap} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -4 & 3 & -3 & 1 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ vf \\ a \\ gap \end{bmatrix}$$

where  $v, a$  is the velocity and acceleration of this car,  $vf$  is the velocity of car in front, and  $gap$  is the distance between the two cars.

# Example Contd

The transpose matrix  $A^T$  has one negative real eigenvalue  $\lambda$ .

If  $\vec{r} = [r1, r2, r3, r4]^T$  is the eigenvector corresponding to  $\lambda$ , then consider the polynomial

$$p = r1 * v + r2 * vf + r3 * a + r4 * gap$$

Why is this special?

$$\begin{aligned}\frac{dp}{dt} &= \frac{d}{dt}([v, vf, a, gap]\vec{r}) \\ &= (A[v, vf, a, gap])^T \vec{r} \\ &= [v, vf, a, gap] A^T \vec{r} \\ &= [v, vf, a, gap] \lambda \vec{r} = \lambda p\end{aligned}$$

# Progress: Results

Interesting consequences of this observation:

- Can do non-trivial reachability computation for linear systems with mixed eigenvalues
  - existing decidability results can not handle this class of systems
  - several systems in the V2V challenge problems can be handled using this new technique
  - we extract **as much** information from the system **as available**, and bridge the gap between the decidable and undecidable problems

# Progress: Results

Additional interesting consequence:

- Do not need to explicitly compute the real eigenvalues or the eigenvector  $\vec{r}$ 
  - the eigenvalue and eigenvector are easily seen to be **algebraic**
  - symbolic decision procedures for real closed fields can handle the algebraic expressions representing these eigenvalues
- Although the new idea applies specifically only to linear systems, it suggests ways to handle **non-linear** systems as well



# Whats in the Demo

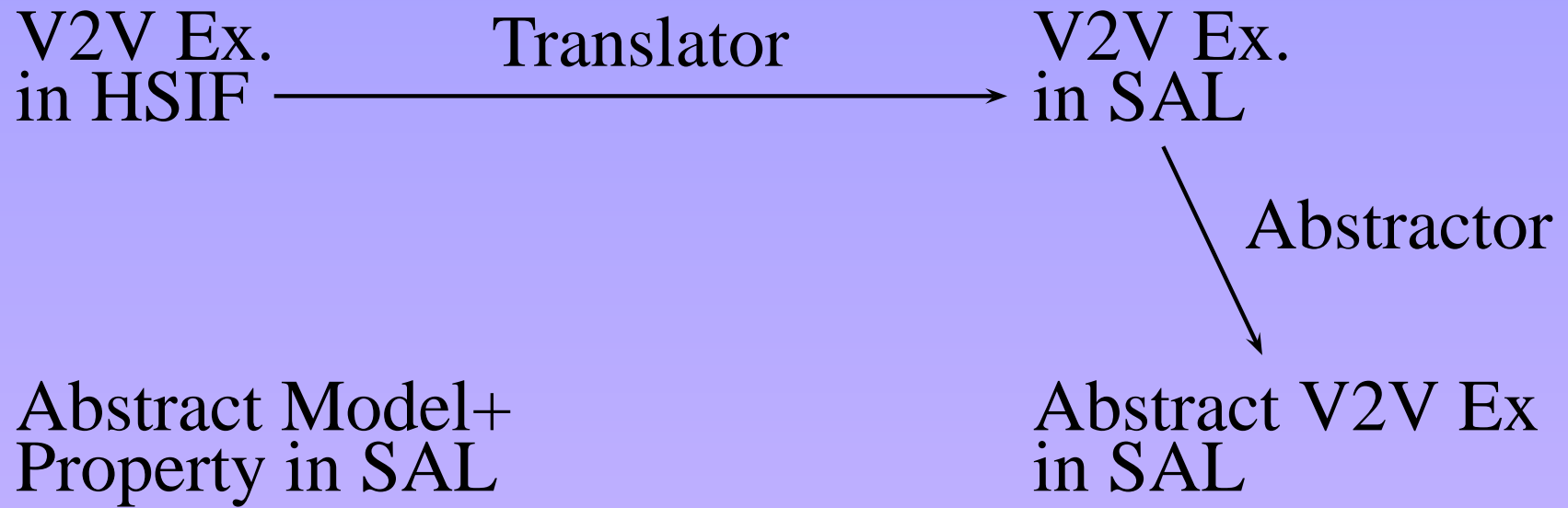
V2V Ex. in HSIF  $\xrightarrow{\text{Translator}}$  V2V Ex. in SAL

Abstract Model+  
Property in SAL

Abstract V2V Ex  
in SAL

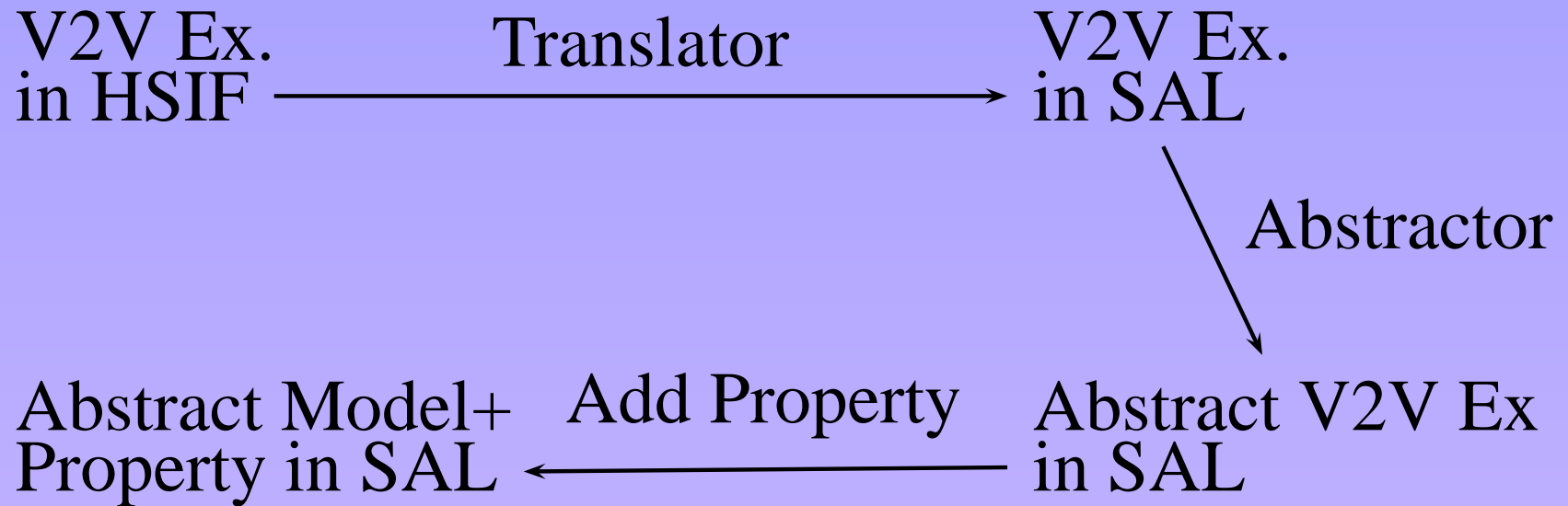
Analysis Result

# Whats in the Demo



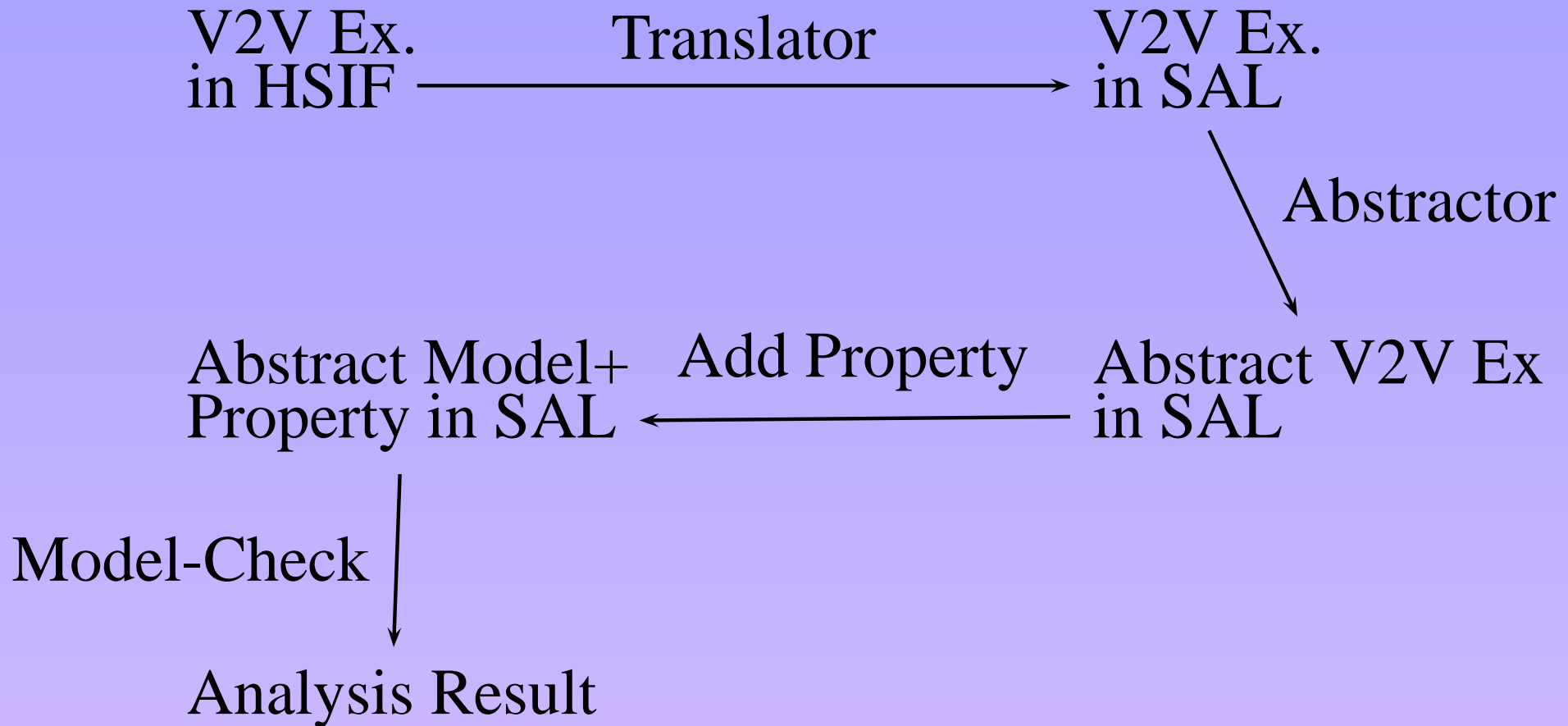
Analysis Result

# Whats in the Demo



Analysis Result

# Whats in the Demo



# Interpreting the Demo Results

- We do **reachability** analysis to show safety, and not just **stability** analysis of the given system
- We prove that the rear car would **not collide** with the car in front **only assuming**
  - a bound on the cars acceleration and deceleration
  - initial state of the two cars falls inside the assumed algebraic **set**
  - the leading car is moving at a constant, but **unspecified**, velocity
- Further analysis can be carried out using the same tools with different initial conditions and different properties

# Project Status: Accomplishments

- Developed a translator from HSIF **into** SAL
- Used the translator to convert a HSIF specification of a V2V cruise control example into SAL
- Automatically abstracted the SAL specification to a simpler SAL specification
- Model-checked the abstraction for safety properties
- Proved collision avoidance for many different controllers developed by the OEP

## Publication:

- “Invisible formal methods for embedded control systems”, To appear in Proceedings of the IEEE
- Most of the new work is unpublished as yet

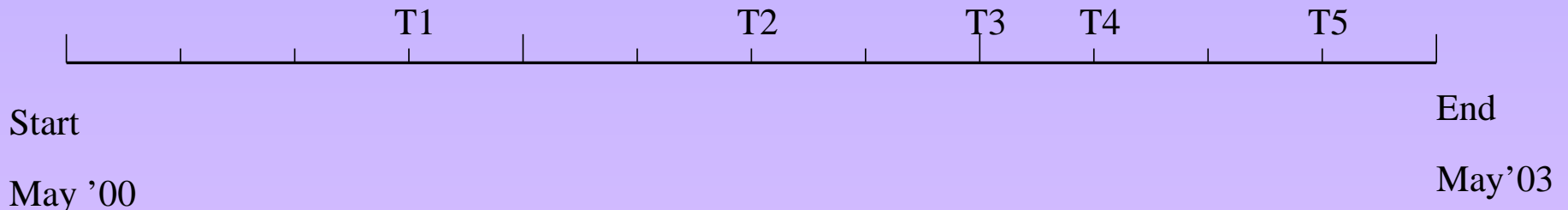
# Project Plans

For the next 6 months

- Develop tools to perform **lightweight formal analysis** of large models specified in, say Simulink/Stateflow
- Develop new insights to perform analysis of **non-linear systems** by generalizing some of the approaches we developed for linear systems
- Build more features into the abstraction tool to handle **compositions** of hybrid automata automatically
- Continue **experimental work** by taking a bigger example from the V2V OEP with many different modes and **non-trivial mode transitions**

# Project Schedule and Milestones

- T1** . Semantics of Stateflow and checking Stateflow diagrams for simple properties
- T2** . Invariant checking and typechecking for SAL specifications of hybrid system models
- T3** . Model-checking tools to explore state-space of abstracted systems
- T4** . Abstraction technique enhancements and composition
- T5** . Interface of tools with other tools and Simulink/Stateflow and further experimentation





# Project Schedule

- Development of analysis tools has been an incremental process—new techniques were (and are being) implemented and tested on challenge problems
- Focus on analysis has continued longer than initially expected
- Revisiting the lightweight methods to connect earlier work and some new work with the Matlab tools: given the availability of translators developed by other Mobies participants

# Technology Transition/Transfer

- SAL/PVS integration with decision procedure for real closed fields to ICASE for verification of aircraft collision avoidance algorithms
- Rockwell-Collins considering SAL to integrate their various development and analysis tools
- BioSpice program