Confluence Properties on Open Terms in the First-Order Theory of Rewriting

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joint work with

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IWC 2016, Obergurgl, September 8

FORT

property → decision mode ← TRS

↓

yes | no | ?

FORT

property → synthesis mode → TRS

no | ?

FORT

property → ← → TRS

$$\forall s\, \exists t\, (s \to^* t \wedge \neg \exists u\, (t \to u) \implies \exists v\, (s \mathrel{+\!\!\!\!\to} v \vee v \xrightarrow{\epsilon} t))$$

yes | no | ?

FORT

property $\longrightarrow$ FORT $\longleftrightarrow$ TRS

$$\forall s\, \exists t\, (s \to^* t \wedge \neg \exists u\, (t \to u)) \implies \exists v\, (s \not\twoheadrightarrow v \vee v \xrightarrow{\epsilon} t))$$

$\downarrow$

yes | no | ?

FORT is based on tree automata techniques (Dauchet and Tison, LICS 1990)

Outline

- First-Order Theory of Rewriting
- Automation
- Properties on Open Terms
- Experiments
- Future Work

First-Order Theory of Rewriting

- first-order logic over a language $\mathcal{L}$ without function symbols

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Derived Predicates

$$s \to^* t \iff s \to^+ t \lor s = t \qquad s \leftrightarrow t \iff s \to t \lor t \to s$$

$$s \to^! t \iff s \to^* t \land \neg \exists\, u \,(t \to u) \qquad s \downarrow t \iff \exists\, u \,(s \to^* u \land t \to^* u)$$

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$$\mathrm{CR}(t) \iff \forall\, u\, \forall\, v\, (t \to^* u \wedge t \to v \implies u \downarrow v)$$

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$$\mathrm{WCR}(t) \iff \forall\, u\, \forall\, v\, (t \to u \land t \to v \implies u \downarrow v) \qquad \mathrm{WCR} \iff \forall\, t\; \mathrm{WCR}(t)$$

$$\mathrm{WN}(t) \iff \exists\, u\, (t \to^! u) \qquad \mathrm{WN} \iff \forall\, t\; \mathrm{WN}(t)$$

$$\mathrm{UN}(t) \iff \forall\, u\, \forall\, v\, (t \to^! u \land t \to^! v \implies u = v) \qquad \mathrm{UN} \iff \forall\, t\; \mathrm{UN}(t)$$

$$\mathrm{NFP}(t) \iff \forall\, u\, \forall\, v\, (t \to u \land t \to^! v \implies u \to^! v) \qquad \mathrm{NFP} \iff \forall\, t\; \mathrm{NFP}(t)$$

$$\mathrm{NF}(t) \iff \neg \exists\, u\, (t \to u)$$

$$\mathrm{UNC} \iff \forall\, t\, \forall\, u\, (t \leftrightarrow^* u \land \mathrm{NF}(t) \land \mathrm{NF}(u) \implies t = u)$$

Outline

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    - ground tree transducers (GTTs) for $\nrightarrow$
    - RR_n automata for all relations

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Remark

formulas are not transformed into prenex normal form, since this increases the dimension of involved relations

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FSCD 2016 submission

It should be stressed that the above properties are restricted to ground terms. So CR *stands for ground-confluence, which is different from confluence, even in the presence of ground terms; consider e.g. the rules* $f(x) \to x$ *and* $f(x) \to c$.

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Example

TRS

$$\mathsf{a} \to \mathsf{b} \qquad \mathsf{f}(\mathsf{a}, x) \to \mathsf{b} \qquad \mathsf{f}(\mathsf{b}, \mathsf{b}) \to \mathsf{b}$$

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Confluence Related Properties

$$
\begin{aligned}
\mathsf{CR}: &\quad \forall s\, \forall t\, \forall u\, (s \to^* t \wedge s \to u \implies t \downarrow u) \\
\mathsf{WCR}: &\quad \forall s\, \forall t\, \forall u\, (s \to t \wedge s \to u \implies t \downarrow u) \\
\mathsf{UN}: &\quad \forall s\, \forall t\, \forall u\, (s \to^! t \wedge s \to^! u \implies t = u) \\
\mathsf{UNC}: &\quad \forall t\, \forall u\, (t \leftrightarrow^* u \wedge \mathsf{NF}(t) \wedge \mathsf{NF}(u) \implies t = u) \\
\mathsf{NFP}: &\quad \forall s\, \forall t\, \forall u\, (s \to t \wedge s \to^! u \implies t \to^! u) \\
\mathsf{SCR}: &\quad \forall s\, \forall t\, \forall u\, (s \to t \wedge s \to u \implies \exists v\, (t \to^= v \wedge u \to^* v))
\end{aligned}
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\end{aligned}$$

$$\mathfrak{P} = \{\, \mathrm{CR}, \mathrm{WCR}, \mathrm{UN}, \mathrm{UNC}, \mathrm{NFP}, \mathrm{SCR} \,\}$$

Relationships

$$\begin{tikzcd}
\mathsf{WCR} & & \mathsf{CR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \mathsf{NFP} \arrow[r, Rightarrow] & \mathsf{UNC} \arrow[r, Rightarrow] & \mathsf{UN} \\
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Notation

$\textcolor{red}{\mathsf{G}}P$ denotes property P restricted to ground terms

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GP denotes property P restricted to ground terms

Remark

$\forall\, P \in \mathfrak{P} \quad GP \not\Rightarrow P$

Lemma

$\forall$ *left-linear right-ground* TRS $(\mathcal{F}, \mathcal{R})$

1. $(\mathcal{F}, \mathcal{R}) \vDash P \iff (\mathcal{F} \cup \{\, c \,\}, \mathcal{R}) \vDash \mathsf{G}P \quad \forall\, P \in \mathfrak{P} \setminus \{\, \mathsf{UNC} \,\}$

with fresh constant c

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2. $(\mathcal{F},\mathcal{R}) \vDash \mathsf{UNC} \iff (\mathcal{F} \cup \{\, c, c' \,\},\mathcal{R}) \vDash \mathsf{GUNC}$

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with fresh constants c *and* c'

Example

left-linear right-ground TRS

$$\mathsf{a} \to \mathsf{b} \qquad \mathsf{f}(x,\mathsf{a}) \to \mathsf{f}(\mathsf{b},\mathsf{b}) \qquad \mathsf{f}(\mathsf{b},x) \to \mathsf{f}(\mathsf{b},\mathsf{b}) \qquad \mathsf{f}(\mathsf{f}(x,y),z) \to \mathsf{f}(\mathsf{b},\mathsf{b})$$

- does not satisfy UNC: $\mathsf{f}(x,\mathsf{b}) \leftarrow \mathsf{f}(x,\mathsf{a}) \to \mathsf{f}(\mathsf{b},\mathsf{b}) \leftarrow \mathsf{f}(y,\mathsf{a}) \to \mathsf{f}(y,\mathsf{b})$

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2. $(\mathcal{F}, \mathcal{R}) \vDash \mathsf{UNC} \quad \Longleftrightarrow \quad (\mathcal{F} \cup \{\, c, c' \,\}, \mathcal{R}) \vDash \mathsf{GUNC}$

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- adding single fresh constant c is not enough to violate GUNC

Lemma

$\forall$ *left-linear right-ground* TRS $(\mathcal{F}, \mathcal{R})$

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2. $(\mathcal{F}, \mathcal{R}) \vDash \mathsf{UNC} \iff (\mathcal{F} \cup \{ c, c' \}, \mathcal{R}) \vDash \mathsf{GUNC}$

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- adding single fresh constant c is not enough to violate GUNC
- GUNC is violated by adding another fresh constant c':

$$\mathsf{f}(\mathsf{c}, \mathsf{b}) \leftarrow \mathsf{f}(\mathsf{c}, \mathsf{a}) \to \mathsf{f}(\mathsf{b}, \mathsf{b}) \leftarrow \mathsf{f}(\mathsf{c}', \mathsf{a}) \to \mathsf{f}(\mathsf{c}', \mathsf{b})$$

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  but GP does not hold after adding additional constants c and c'

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- adding fresh unary function symbol g and fresh constant c in order to create infinitely many ground normal forms

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  but $\mathrm{G}P$ does not hold after adding additional constants c and c'
- adding fresh unary function symbol g and fresh constant c in order to create infinitely many ground normal forms is unsound in general:

  consider $\mathsf{a} \to \mathsf{b}$ and $\forall\, s\,\forall\, t\;(s \to t \implies s \xrightarrow{\epsilon} t)$

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Lemma

$\forall$ *left-linear right-ground* TRS $(\mathcal{F}, \mathcal{R})$ *such that* *$\mathcal{R}$ is ground or $\mathcal{F}$ is monadic*

$$(\mathcal{F}, \mathcal{R}) \vDash P \iff (\mathcal{F}, \mathcal{R}) \vDash \mathsf{G}P \qquad \forall\, P \in \mathfrak{P}$$

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Example

checking GCR of TRS

$$\mathsf{f}(\mathsf{f}(\mathsf{f}(x))) \to \mathsf{a} \qquad \mathsf{f}(\mathsf{f}(\mathsf{a})) \to \mathsf{a} \qquad \mathsf{f}(\mathsf{a}) \to \mathsf{a}$$
$$\mathsf{f}(\mathsf{f}(\mathsf{g}(\mathsf{g}(x)))) \to \mathsf{f}(\mathsf{a}) \qquad \mathsf{g}(\mathsf{f}(\mathsf{a})) \to \mathsf{a} \qquad \mathsf{g}(\mathsf{a}) \to \mathsf{a}$$

takes 0.85 seconds but 1.73 seconds if fresh constant is added

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& \mathsf{GSCR} \arrow[dr, Rightarrow] & & & & \\
\mathsf{GWCR} & & \mathsf{GCR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \mathsf{GNFP} \arrow[r, Rightarrow] & \mathsf{GUNC} \arrow[r, Rightarrow] & \mathsf{GUN}
\end{tikzcd}$$

Relationships

$$
\begin{tikzcd}
\mathsf{WCR} \arrow[d, Rightarrow] & & \mathsf{CR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathsf{NFP} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathsf{UNC} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathsf{UN} \arrow[ddd, Rightarrow] \\
 & \mathsf{SCR} \arrow[ur, Rightarrow] \arrow[d, Rightarrow] & & & & \\
 & \mathsf{GSCR} \arrow[dr, Rightarrow] & & & & \\
\mathsf{GWCR} & & \mathsf{GCR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \mathsf{GNFP} \arrow[r, Rightarrow] & \mathsf{GUNC} \arrow[r, Rightarrow] & \mathsf{GUN}
\end{tikzcd}
$$

Synthesis Experiments with **FORT 0.2**

```
-S -f "a:0 b:0 f:2"
```

Relationships

$$
\begin{tikzcd}
\mathrm{WCR} \arrow[d, Rightarrow] & & \mathrm{CR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{NFP} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{UNC} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{UN} \arrow[ddd, Rightarrow] \\
& \mathrm{SCR} \arrow[ur, Rightarrow] \arrow[d, Rightarrow] & & & & \\
& \mathrm{GSCR} \arrow[dr, Rightarrow] & & & & \\
\mathrm{GWCR} & & \mathrm{GCR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \mathrm{GNFP} \arrow[r, Rightarrow] & \mathrm{GUNC} \arrow[r, Rightarrow] & \mathrm{GUN}
\end{tikzcd}
$$

Synthesis Experiments with **FORT 0.2**

```
-S -f "a:0 b:0 f:2"
```

```
"GWCR & ~WCR & ~GCR"
```

Relationships

$$
\begin{tikzcd}
\mathsf{WCR} \arrow[dddd, Rightarrow] & & \mathsf{CR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] \arrow[dddd, Rightarrow] & \mathsf{NFP} \arrow[r, Rightarrow] \arrow[dddd, Rightarrow] & \mathsf{UNC} \arrow[r, Rightarrow] \arrow[dddd, Rightarrow] & \mathsf{UN} \arrow[dddd, Rightarrow] \\
 & \mathsf{SCR} \arrow[ur, Rightarrow] \arrow[d, Rightarrow] & & & & \\
 & \mathsf{GSCR} \arrow[ddr, Rightarrow] & & & & \\
 & & & & & \\
\mathsf{GWCR} & & \mathsf{GCR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \mathsf{GNFP} \arrow[r, Rightarrow] & \mathsf{GUNC} \arrow[r, Rightarrow] & \mathsf{GUN}
\end{tikzcd}
$$

Synthesis Experiments with **FORT 0.2**

```
-S -f "a:0 b:0 f:2"
```

`"GWCR & ~WCR & ~GCR"` $\mathsf{a} \to \mathsf{b}$ $\mathsf{f}(x, \mathsf{a}) \to \mathsf{a}$ $\mathsf{a} \to \mathsf{f}(\mathsf{a}, \mathsf{a})$ $80\,s$

Relationships

$$
\begin{tikzcd}
\mathrm{WCR} \arrow[ddd, Rightarrow] & & \mathrm{CR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{NFP} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{UNC} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{UN} \arrow[ddd, Rightarrow] \\
& \mathrm{SCR} \arrow[ur, Rightarrow] \arrow[d, Rightarrow] & & & & \\
& \mathrm{GSCR} \arrow[dr, Rightarrow] & & & & \\
\mathrm{GWCR} & & \mathrm{GCR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \mathrm{GNFP} \arrow[r, Rightarrow] & \mathrm{GUNC} \arrow[r, Rightarrow] & \mathrm{GUN}
\end{tikzcd}
$$

Synthesis Experiments with **FORT 0.2**

```
-S -f "a:0 b:0 f:2"
```

`"GWCR & ~WCR & ~GCR"` $\mathsf{a} \to \mathsf{b}$ $\mathsf{f}(x,\mathsf{a}) \to \mathsf{a}$ $\mathsf{a} \to \mathsf{f}(\mathsf{a},\mathsf{a})$ 80 s

`"GCR & ~CR & ~GSCR"`

Relationships

$$
\begin{tikzcd}
\mathsf{WCR} \arrow[ddd, Rightarrow] & & \mathsf{CR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathsf{NFP} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathsf{UNC} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathsf{UN} \arrow[ddd, Rightarrow] \\
& \mathsf{SCR} \arrow[ur, Rightarrow] \arrow[d, Rightarrow] & & & & \\
& \mathsf{GSCR} \arrow[dr, Rightarrow] & & & & \\
\mathsf{GWCR} & & \mathsf{GCR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \mathsf{GNFP} \arrow[r, Rightarrow] & \mathsf{GUNC} \arrow[r, Rightarrow] & \mathsf{GUN}
\end{tikzcd}
$$

Synthesis Experiments with **FORT 0.2**

`-S -f "a:0 b:0 f:2"`

`"GWCR & ~WCR & ~GCR"`	$\mathsf{a} \to \mathsf{b}$	$\mathsf{f}(x,\mathsf{a}) \to \mathsf{a}$	$\mathsf{a} \to \mathsf{f}(\mathsf{a},\mathsf{a})$	80 s
`"GCR & ~CR & ~GSCR"`	$\mathsf{a} \to \mathsf{b}$	$\mathsf{f}(x,\mathsf{a}) \to \mathsf{b}$	$\mathsf{b} \to \mathsf{f}(\mathsf{a},\mathsf{a})$	109 s

Relationships

$$
\begin{tikzcd}
\mathsf{WCR} \arrow[ddd, Rightarrow] & & \mathsf{CR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathsf{NFP} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathsf{UNC} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathsf{UN} \arrow[ddd, Rightarrow] \\
& \mathsf{SCR} \arrow[ur, Rightarrow] \arrow[d, Rightarrow] & & & & \\
& \mathsf{GSCR} \arrow[dr, Rightarrow] & & & & \\
\mathsf{GWCR} & & \mathsf{GCR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \mathsf{GNFP} \arrow[r, Rightarrow] & \mathsf{GUNC} \arrow[r, Rightarrow] & \mathsf{GUN}
\end{tikzcd}
$$

Synthesis Experiments with **FORT 0.2**

```
-S -f "a:0 b:0 f:2"
```

`"GWCR & ~WCR & ~GCR"`	$\mathsf{a} \to \mathsf{b}$	$\mathsf{f}(x,\mathsf{a}) \to \mathsf{a}$	$\mathsf{a} \to \mathsf{f}(\mathsf{a},\mathsf{a})$	80 s
`"GCR & ~CR & ~GSCR"`	$\mathsf{a} \to \mathsf{b}$	$\mathsf{f}(x,\mathsf{a}) \to \mathsf{b}$	$\mathsf{b} \to \mathsf{f}(\mathsf{a},\mathsf{a})$	109 s
`"GNFP & ~NFP & ~GCR"`				

Relationships

$$\begin{tikzcd}
\mathrm{WCR} \arrow[ddd, Rightarrow] & & \mathrm{CR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{NFP} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{UNC} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{UN} \arrow[ddd, Rightarrow] \\
& \mathrm{SCR} \arrow[ur, Rightarrow] \arrow[d, Rightarrow] & & & & \\
& \mathrm{GSCR} \arrow[dr, Rightarrow] & & & & \\
\mathrm{GWCR} & & \mathrm{GCR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \mathrm{GNFP} \arrow[r, Rightarrow] & \mathrm{GUNC} \arrow[r, Rightarrow] & \mathrm{GUN}
\end{tikzcd}$$

Synthesis Experiments with **FORT 0.2**

```
-S -f "a:0 b:0 f:2"
```

`"GWCR & ~WCR & ~GCR"`	$a \to b$	$f(x, a) \to a$	$a \to f(a, a)$	80 s
`"GCR & ~CR & ~GSCR"`	$a \to b$	$f(x, a) \to b$	$b \to f(a, a)$	109 s
`"GNFP & ~NFP & ~GCR"`	$a \to b$	$f(x, a) \to f(a, a)$	$f(b, b) \to f(a, a)$	16 s

Relationships

$$
\begin{tikzcd}
\mathrm{WCR} \arrow[ddd, Rightarrow] & & \mathrm{CR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{NFP} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{UNC} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{UN} \arrow[ddd, Rightarrow] \\
& \mathrm{SCR} \arrow[ur, Rightarrow] \arrow[d, Rightarrow] & & & & \\
& \mathrm{GSCR} \arrow[dr, Rightarrow] & & & & \\
\mathrm{GWCR} & & \mathrm{GCR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \mathrm{GNFP} \arrow[r, Rightarrow] & \mathrm{GUNC} \arrow[r, Rightarrow] & \mathrm{GUN}
\end{tikzcd}
$$

Synthesis Experiments with **FORT 0.2**

```
-S -f "a:0 b:0 f:2"
```

"GWCR & ~WCR & ~GCR"	$\mathsf{a} \to \mathsf{b}$	$\mathsf{f}(x,\mathsf{a}) \to \mathsf{a}$		$\mathsf{a} \to \mathsf{f}(\mathsf{a},\mathsf{a})$	80 s
"GCR & ~CR & ~GSCR"	$\mathsf{a} \to \mathsf{b}$	$\mathsf{f}(x,\mathsf{a}) \to \mathsf{b}$		$\mathsf{b} \to \mathsf{f}(\mathsf{a},\mathsf{a})$	109 s
"GNFP & ~NFP & ~GCR"	$\mathsf{a} \to \mathsf{b}$	$\mathsf{f}(x,\mathsf{a}) \to \mathsf{f}(\mathsf{a},\mathsf{a})$	$\mathsf{f}(\mathsf{b},\mathsf{b}) \to \mathsf{f}(\mathsf{a},\mathsf{a})$		16 s
"GUNC & ~UNC & ~GNFP"					

Relationships

$$
\begin{tikzcd}
\mathsf{WCR} \arrow[ddd, Rightarrow] & & \mathsf{CR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathsf{NFP} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \color{red}{\mathsf{UNC}} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow, red] & \mathsf{UN} \arrow[ddd, Rightarrow] \\
& \mathsf{SCR} \arrow[ur, Rightarrow] \arrow[d, Rightarrow] & & & & \\
& \mathsf{GSCR} \arrow[dr, Rightarrow] & & & & \\
\mathsf{GWCR} & & \mathsf{GCR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \color{red}{\mathsf{GNFP}} \arrow[r, Rightarrow, red] & \color{red}{\mathsf{GUNC}} \arrow[r, Rightarrow] & \mathsf{GUN}
\end{tikzcd}
$$

Synthesis Experiments with **FORT 0.2**

```
-S -f "a:0 b:0 f:2"
```

"GWCR & ~WCR & ~GCR"	$a \to b$	$f(x, a) \to a$	$a \to f(a, a)$	80 s
"GCR & ~CR & ~GSCR"	$a \to b$	$f(x, a) \to b$	$b \to f(a, a)$	109 s
"GNFP & ~NFP & ~GCR"	$a \to b$	$f(x, a) \to f(a, a)$	$f(b, b) \to f(a, a)$	16 s
"GUNC & ~UNC & ~GNFP"	$a \to a$	$f(x, a) \to a$	$f(b, x) \to b$	95 s

Relationships

$$
\begin{tikzcd}
\mathrm{WCR} & & \mathrm{CR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{NFP} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{UNC} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{UN} \arrow[ddd, Rightarrow] \\
& \mathrm{SCR} \arrow[ur, Rightarrow] \arrow[d, Rightarrow] & & & & \\
& \mathrm{GSCR} \arrow[dr, Rightarrow] & & & & \\
\mathrm{GWCR} \arrow[from=uuu, Rightarrow] & & \mathrm{GCR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \mathrm{GNFP} \arrow[r, Rightarrow] & \mathrm{GUNC} \arrow[r, Rightarrow] & \mathrm{GUN}
\end{tikzcd}
$$

Synthesis Experiments with **FORT 0.2**

```
-S -f "a:0 b:0 f:2"
```

"GWCR & ~WCR & ~GCR"	$a \to b$	$f(x,a) \to a$	$a \to f(a,a)$	80 s
"GCR & ~CR & ~GSCR"	$a \to b$	$f(x,a) \to b$	$b \to f(a,a)$	109 s
"GNFP & ~NFP & ~GCR"	$a \to b$	$f(x,a) \to f(a,a)$	$f(b,b) \to f(a,a)$	16 s
"GUNC & ~UNC & ~GNFP"	$a \to a$	$f(x,a) \to a$	$f(b,x) \to b$	95 s

Relationships

$$
\begin{tikzcd}
\mathrm{WCR} \arrow[ddd, Rightarrow] & & \mathrm{CR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{NFP} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{UNC} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathrm{UN} \arrow[ddd, Rightarrow] \\
& \mathrm{SCR} \arrow[ur, Rightarrow] \arrow[d, Rightarrow] & & & & \\
& \mathrm{GSCR} \arrow[dr, Rightarrow] & & & & \\
\mathrm{GWCR} & & \mathrm{GCR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \mathrm{GNFP} \arrow[r, Rightarrow] & \mathrm{GUNC} \arrow[r, Rightarrow] & \mathrm{GUN}
\end{tikzcd}
$$

Synthesis Experiments with **FORT 0.2**

```
-S -f "a:0 b:0 f:2"
```

`"GWCR & ~WCR & ~GCR"`	$a \to b$	$f(x, a) \to a$		$a \to f(a, a)$	$80\,s$
`"GCR & ~CR & ~GSCR"`	$a \to b$	$f(x, a) \to b$		$b \to f(a, a)$	$109\,s$
`"GNFP & ~NFP & ~GCR"`	$a \to b$	$f(x, a) \to f(a, a)$		$f(b, b) \to f(a, a)$	$16\,s$
`"~GNFP & GUNC & ~UNC"`	$a \to a$	$f(x, a) \to a$	$f(b, x) \to b$		$21\,s$

Relationships

$$
\begin{tikzcd}
\mathsf{WCR} \arrow[ddd, Rightarrow] & & \mathsf{CR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathsf{NFP} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathsf{UNC} \arrow[r, Rightarrow] \arrow[ddd, Rightarrow] & \mathsf{UN} \arrow[ddd, Rightarrow] \\
& \mathsf{SCR} \arrow[ur, Rightarrow] \arrow[d, Rightarrow] & & & & \\
& \mathsf{GSCR} \arrow[dr, Rightarrow] & & & & \\
\mathsf{GWCR} & & \mathsf{GCR} \arrow[ll, Rightarrow] \arrow[r, Rightarrow] & \mathsf{GNFP} \arrow[r, Rightarrow] & \mathsf{GUNC} \arrow[r, Rightarrow] & \mathsf{GUN}
\end{tikzcd}
$$

Synthesis Experiments with FORT 1.0

```
-S -f "a:0 b:0 f:2"
```

"GWCR & ~WCR & ~GCR"	$a \to b$	$f(x,a) \to a$		$a \to f(a,a)$	8 - 20 s
"GCR & ~CR & ~GSCR"	$a \to b$	$f(x,a) \to b$		$b \to f(a,a)$	8 - 10 s
"GNFP & ~NFP & ~GCR"	$a \to b$	$f(x,a) \to f(a,a)$		$f(b,b) \to f(a,a)$	9 - 11 s
"~GNFP & GUNC & ~UNC"	$a \to a$	$f(x,a) \to a$		$f(b,x) \to b$	2 - 4 s

Example

"GUN & ~UN & ~GUNC"

$$\begin{array}{ccc} \mathsf{b} \to \mathsf{a} & \mathsf{d} \to \mathsf{c} & \mathsf{f}(x,\mathsf{e}) \to \mathsf{A} \\ \mathsf{b} \to \mathsf{c} & \mathsf{d} \to \mathsf{e} & \mathsf{f}(x,\mathsf{A}) \to \mathsf{A} \\ \mathsf{c} \to \mathsf{c} & \mathsf{f}(x,\mathsf{a}) \to \mathsf{A} & \mathsf{f}(\mathsf{c},x) \to \mathsf{A} \end{array}$$

Example

`"GUN & ~UN & ~GUNC"`

$$\begin{array}{ccc} \mathsf{b} \to \mathsf{a} & \mathsf{d} \to \mathsf{c} & \mathsf{f}(x, \mathsf{e}) \to \mathsf{A} \\ \mathsf{b} \to \mathsf{c} & \mathsf{d} \to \mathsf{e} & \mathsf{f}(x, \mathsf{A}) \to \mathsf{A} \\ \mathsf{c} \to \mathsf{c} & \mathsf{f}(x, \mathsf{a}) \to \mathsf{A} & \mathsf{f}(\mathsf{c}, x) \to \mathsf{A} \end{array}$$

Comparison (GCR)

AGCP is recent tool for ground-confluence of many-sorted TRSs based on rewriting induction (Aoto and Toyama, FSCD 2016)

Example

"GUN & ~UN & ~GUNC"

$$\begin{array}{ccc} \mathsf{b} \to \mathsf{a} & \mathsf{d} \to \mathsf{c} & \mathsf{f}(x, \mathsf{e}) \to \mathsf{A} \\ \mathsf{b} \to \mathsf{c} & \mathsf{d} \to \mathsf{e} & \mathsf{f}(x, \mathsf{A}) \to \mathsf{A} \\ \mathsf{c} \to \mathsf{c} & \mathsf{f}(x, \mathsf{a}) \to \mathsf{A} & \mathsf{f}(\mathsf{c}, x) \to \mathsf{A} \end{array}$$

Comparison (GCR)

AGCP is recent tool for ground-confluence of many-sorted TRSs based on rewriting induction (Aoto and Toyama, FSCD 2016)

65 TRSs	AGCP	(∅ time)	FORT 0.2	(∅ time)
yes	8	(0.02 s)	42	(0.42 s)
no	–		14	(3.88 s)
maybe	56	(0.19 s)	–	
timeout	1		9	
total time	71 s		612 s	

Example

"GUN & ~UN & ~GUNC"

$$\begin{array}{ccc} \mathsf{b} \to \mathsf{a} & \mathsf{d} \to \mathsf{c} & \mathsf{f}(x,\mathsf{e}) \to \mathsf{A} \\ \mathsf{b} \to \mathsf{c} & \mathsf{d} \to \mathsf{e} & \mathsf{f}(x,\mathsf{A}) \to \mathsf{A} \\ \mathsf{c} \to \mathsf{c} & \mathsf{f}(x,\mathsf{a}) \to \mathsf{A} & \mathsf{f}(\mathsf{c},x) \to \mathsf{A} \end{array}$$

Comparison (GCR)

AGCP is recent tool for ground-confluence of many-sorted TRSs based on rewriting induction (Aoto and Toyama, FSCD 2016)

65 TRSs	AGCP	(∅ time)	FORT 0.2	(∅ time)	FORT 1.0	(∅ time)
yes	8	(0.02 s)	42	(0.42 s)	43	(0.26 s)
no	–		14	(3.88 s)	18	(0.96 s)
maybe	56	(0.19 s)	–		–	
timeout	1		9		4	
total time	71 s		612 s		268 s	

Example

"GUN & ~UN & ~G

Let's see what happens at

$$\mathsf{f}(x, \mathsf{e}) \to \mathsf{A}$$
$$\mathsf{f}(x, \mathsf{A}) \to \mathsf{A}$$
$$\mathsf{f}(\mathsf{c}, x) \to \mathsf{A}$$

Comparison (GCR)

AGCP is recent tool for ground-confluence of many-sorted TRSs based on rewriting induction (Aoto and Toyama, FSCD 2016)

65 TRSs	AGCP	(∅ time)	FORT 0.2	(∅ time)	FORT 1.0	(∅ time)
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timeout	1		9		4	
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Outline

Limitation

restriction to left-linear right-ground TRSs is hard to overcome because first-order theory of one-step rewriting ($\rightarrow$) is undecidable

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- . . . even for linear non-erasing TRSs (Treinen, 1998)

Limitation

restriction to left-linear right-ground TRSs is hard to overcome because first-order theory of one-step rewriting ($\rightarrow$) is undecidable

- ... even for linear non-erasing TRSs (Treinen, 1998)
- ... even for complete right-ground TRSs (Vorobyov, 2002)

Limitation

restriction to left-linear right-ground TRSs is hard to overcome because first-order theory of one-step rewriting ($\to$) is undecidable

- ... even for linear non-erasing TRSs (Treinen, 1998)
- ... even for complete right-ground TRSs (Vorobyov, 2002)

Ongoing and Future Work

- generating witnesses for existential formulas and formulas with free variables

Limitation

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Ongoing and Future Work

- generating witnesses for existential formulas and formulas with free variables
- support for combinations of TRSs (e.g., to express commutation)

Limitation

restriction to left-linear right-ground TRSs is hard to overcome because first-order theory of one-step rewriting ($\to$) is undecidable

- ... even for linear non-erasing TRSs (Treinen, 1998)
- ... even for complete right-ground TRSs (Vorobyov, 2002)

Ongoing and Future Work

- generating witnesses for existential formulas and formulas with free variables
- support for combinations of TRSs (e.g., to express commutation)
- incorporating rewrite strategies

Limitation

restriction to left-linear right-ground TRSs is hard to overcome because first-order theory of one-step rewriting ($\to$) is undecidable

- ... even for linear non-erasing TRSs (Treinen, 1998)
- ... even for complete right-ground TRSs (Vorobyov, 2002)

Ongoing and Future Work

- generating witnesses for existential formulas and formulas with free variables
- support for combinations of TRSs (e.g., to express commutation)
- incorporating rewrite strategies
- formalizing underlying theory in Isabelle/HOL