Exercise 1 : Curien-Herbelin-Wadler's calculus

We consider a special type $\perp$ and a continuation constant top of type $\perp$. Let $\neg A$ be an abbreviation for $A{\rightarrow}\perp$. Let A and B be simple types.

1. Give a term efq_A such that $\vdash \mathsf{efq}_A : \perp{\rightarrow}A$;

2. Give a term lem_A such that $\vdash \mathsf{lem}_A : A \vee \neg A$;

3. Give a term edn_A such that $\vdash \mathsf{edn}_A : (\neg\neg A){\rightarrow}A$;

4. Give a command c_A^B such that $c_A^B : (x : (A{\rightarrow}B){\rightarrow}A) \vdash \beta : A)$

5. Give a term pl_A^B such that $x : (A{\rightarrow}B){\rightarrow}A) \vdash \mathsf{pl}_A^B : A$;

Exercise 2 : Programming in the simply-typed λ-calculus

1. Let a be a simple atomic type and $\Gamma = (x : a, f : a{\rightarrow}a)$.
   Describe all λ-terms in normal form that are of type a in Γ.

2. Let $\underline{n}$ be Church's numeral n.

   (a) Write a λ-term suc such that $\mathsf{suc}\ \underline{n} \longrightarrow_\beta^* \underline{n+1}$

   (b) Write a λ-term plus such that $\mathsf{plus}\ \underline{n}\ \underline{q} \longrightarrow_\beta^* \underline{n+q}$

   (c) Write a λ-term times such that $\mathsf{times}\ \underline{n}\ \underline{q} \longrightarrow_\beta^* \underline{n \times q}$

3. Write three λ-terms $\mathsf{true}, \mathsf{false}, \mathsf{if}$ such that $\mathsf{if}\ \mathsf{true}\ M\ N \longrightarrow_\beta^* M$ and $\mathsf{if}\ \mathsf{false}\ M\ N \longrightarrow_\beta^* N$