Exercise 1 : Fixpoints
Consider the following syntax for types:

$$A, B ::= \ a \mid A \vee B \mid A \wedge B \mid A \to B \mid \top \mid \bot \mid \mathsf{Fix}a.A$$

where the last three constructs are the constants true, false, and a fixpoint construct that binds a in A. Types are quotiented by α-renaming, and by the equation $\mathsf{Fix}a.A = \left\{ {}^{\mathsf{Fix}a.A}\!/\!_{a} \right\} A$
Using that syntax for types, we work in Curien-Herbelin-Wadler calculus; we just give ourselves the constant term $\star : \top$ and the constant continuation $\mathsf{top} : \bot$.

1. Let A be an arbitrary type. Give a closed term of type $\top \vee (A \wedge a)$.
2. Give a closed term of type $\mathsf{Fix}a.(\top \vee (A \wedge a))$.
3. Given a term t of type A, give a term of type $\mathsf{Fix}a.(\top \vee (A \wedge a))$ that has t as one of its sub-terms.
4. Assuming A is non-empty, describe infinitely many terms of type $\mathsf{Fix}a.(\top \vee (A \wedge a))$.
5. Let $A\mathsf{List}$ be an abbreviation for $\mathsf{Fix}a.(\top \vee (A \wedge a))$. Give a term $\mathsf{El} : A\mathsf{List}$ representing the empty list and a construct Cons such that $\mathsf{Cons}(t, l) : A\mathsf{List}$ represents the list of head t and of tail l.
6. Assume that you now have a mechanism for raising exceptions: a term constant $\mathsf{Exception} : \bot$. Give a typing derivation for $\langle \mathsf{Exception} \bullet \mathsf{top} \rangle$.
7. Consider the usual reduction system for Curien-Herbelin-Wadler calculus.

   Write a term head that returns the head of a non-empty list:
   i.e. such that $\langle \mathsf{head} \bullet \mathsf{Cons}(t, l) :: e \rangle \longrightarrow^* \langle t \bullet e \rangle$
   and that raises an exception when applied to the empty list $\langle \mathsf{head} \bullet \mathsf{El} :: e \rangle \longrightarrow^* \langle \mathsf{Exception} \bullet \mathsf{top} \rangle$
   Give a typing for head.
8. Similarly, write a term tail that returns the head of a non-empty list:
   i.e. such that $\langle \mathsf{tail} \bullet \mathsf{Cons}(t, l) :: e \rangle \longrightarrow^* \langle l \bullet e \rangle$
   and that raises an exception when applied to the empty list $\langle \mathsf{tail} \bullet \mathsf{El} :: e \rangle \longrightarrow^* \langle \mathsf{Exception} \bullet \mathsf{top} \rangle$
   Give a typing for tail.
9. Let c be a command and y a variable not free in c.

   What are the CBV-reducts and normal forms of $\langle \mathsf{head} \bullet \mathsf{El} :: \mu y.c \rangle$?

   What are the CBN-reducts and normal forms of $\langle \mathsf{head} \bullet \mathsf{El} :: \mu y.c \rangle$?

Exercise 2 : Resolving non-confluence by looking at connectives
Let A, B and C be simple types.

1. Give a command c_A such that $c : (y : A \to B, z : A \to C, z' : A \vdash \alpha : B \wedge C)$
2. Give a continuation e_A such that $y : A \to B, z : A \to C \ ; \ e_A : A \vdash \alpha : B \wedge C$
3. Give a typing for the command $\langle \mathsf{pl}_A^B \bullet e_A \rangle$
   (the term pl_A^B refers to the Exercises of week 1: a term satisfying $x : (A \to B) \to A) \vdash \mathsf{pl}_A^B : A \ ; \ )$
4. What are the reducts and normal forms of that command?
5. Assume $A = A_1 \to A_2$. Can you change pl_A^B or e_A so that $\langle \mathsf{pl}_A^B \bullet e_A \rangle$ is no longer a critical pair (and has the same typing as in question 3)?
6. Assume $A = A_1 \vee A_2$. Can you change pl_A^B or e_A so that $\langle \mathsf{pl}_A^B \bullet e_A \rangle$ is no longer a critical pair (and has the same typing as in question 3)?