CS3202: Logic, Specification and Verification

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Lecture 1 (06/02/2007):

Review of logic, syntax and semantics

Review of propositional logic: syntax

- propositional variables (atoms): P, Q, $\ldots$
- connectives: and $\wedge$, or $\vee$, implies $\Rightarrow$, iff $\Leftrightarrow$, not, $\neg$
- simple context-free grammar of well-formed formulae (wff)
- precedence and associativity (your mileage may vary)

Review of propositional logic: semantics

- yes, models of wffs
- *valuations*: finite maps from the propositional variables of a wff to the Booleans
- extend valuations to all wffs by *structural recursion*: need the truth tables
- a *model* $\mathcal{M}$ of a wff ϕ is a valuation in which ϕ has value true
- tautologies: wffs with value true in *all* valuations

Valuations

- finite maps $\mathcal{V}$, taking propositional variables P, etc. into Booleans

- lift to all wffs by *structural recursion*:

$$
\begin{aligned}
\mathcal{V}(P) &= \text{given}, \\
\mathcal{V}(\phi \wedge \psi) &= tt_{\wedge}(\mathcal{V}(\phi), \mathcal{V}(\psi)) \\
\mathcal{V}(\phi \vee \psi) &= tt_{\vee}(\mathcal{V}(\phi), \mathcal{V}(\psi)) \\
\mathcal{V}(\phi \Rightarrow \psi) &= tt_{\Rightarrow}(\mathcal{V}(\phi), \mathcal{V}(\psi)) \\
\mathcal{V}(\phi \Leftrightarrow \psi) &= tt_{\Leftrightarrow}(\mathcal{V}(\phi), \mathcal{V}(\psi))
\end{aligned}
$$

- where $tt_{\wedge}(v, w)$, etc. are the *truth-table* functions on truth values for the corresponding connective

Logical consequence

- fundamental semantic relation (*consequence relation*) from hypotheses to conclusions

$$\phi_1, \ldots, \phi_n \models \phi$$

- in every valuation in which each of $\phi_1, \ldots, \phi_n$ has value true...
- ...then so too does ϕ
- "entailment", "logical consequence", "semantic consequence" (for propositional logic)

Logical consequence, II

- a *mechanical* process for checking entailments $\phi_1, \ldots, \phi_n \models \phi$
- identify the propositional atoms P, Q ... in $\phi_1, \ldots, \phi_n$, ϕ
- consider all valuations $\mathcal{V}$ on these atoms; (then extend to wffs)
- identify all such $\mathcal{V}$ for which each of $\mathcal{V}(\phi_i)$ = $true$
- check that for all such valuations $\mathcal{V}(\phi)$ = $true$
- Problem: complexity lower bound of 2^p in the number p of propositional atoms

Natural deduction

- *syntactic* consequence relation, capturing "does the conclusion follow from the hypotheses?"

$$\phi_1, \ldots, \phi_n \vdash \phi$$

- defined by *inference rules*: introduction and elimination
- finite system, follows the *structure* of the grammar
- "natural": obviously correct inference from hypothesis to conclusion in each rule
- *soundness*: formalise this "obviously correct" idea
- *completeness*: the rules are, in fact enough to characterise semantic consequence (not at all "obvious")

Questions?