Solving bitvectors with MCSAT: explanations from bits and pieces

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SRI International

IJCAR, July 2020

tl;dl (Too Long; Didn't Listen)

- MCSAT (Model-Constructing Satisfiability)
  is a scheme for SMT-solving (Satisfiability-Modulo-Theories),
  alternative to DPLL($\mathcal{T}$).

- To apply the scheme to a particular theory $\mathcal{T}$,
  you need a form of *interpolation* mechanism for $\mathcal{T}$.

- Designing an efficient mechanism for the full theory of bitvectors is difficult. So we do it for 2 fragments of the theory:
  - Equality + concatenation and extraction of bitvectors
  - A fragment of bitvector arithmetic

  Outside these fragments we use a less efficient, but generic, procedure.

- The approach is implemented in SRI's SMT-solver Yices.

- We experimented it on the SMTLib benchmarks.

1. Overview of MCSAT

The model-constructing approach to SMT-solving 1/2

MCSAT introduced in [dMJ13, JBdM13, Jov17],
inspired by multiple contributions including Conflict Resolution [KTV09] and specific decision procedures for theories such as non-linear arithmetic [JdM12].

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MCSAT tailored to theories with a standard model used for evaluating constraints (example: arithmetic)

Evaluation is a key aspect of MCSAT

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(set of constraints on variables $x_1, \ldots, x_n$)

$=$ finding values for variables $x_1, \ldots, x_n$

(so that constraints evaluate to true)

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MCSAT offers:

- a template for decision procedures
- an integration of such procedures with Boolean reasoning
- new possibilities for combining theories [JBdM13, BGLS19]

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The template is a generalisation of how CDCL works, the core calculus of SAT-solvers.

Run $=$ alternation of search phases and conflict analysis phases

The model-constructing approach to SMT-solving 2/2

- Like CDCL's trail assigns Boolean values to Boolean variables, MCSAT's trail assigns
  - Boolean values to theory atoms; these constitute *theory contraints*
  - model values to first-order variables (e.g., $x \leftarrow 3/4$)

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- If all variables get values while maintaining invariant: **SAT**.
  illustration on the next slide.

Search phase (satisfiable case)

Free var within	Constraints (unit ones in red)	Feasible set	Var
$\{x_1\}$	$C_1^1, \ldots, C_j^1, \ldots$		x_1
$\{x_1, x_2\}$	$C_1^2, C_2^2, \ldots, C_j^2, \ldots$		x_2
$\{x_1, x_2, x_3\}$	$C_1^3, C_2^3, \ldots, C_j^3, \ldots$		x_3
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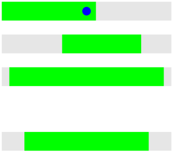

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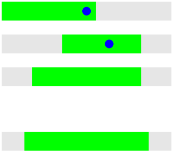

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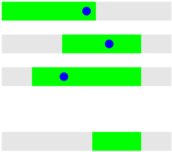

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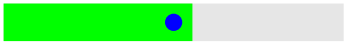
$\{x_1\}$	$C_1^1, \ldots, C_j^1, \ldots$		x_1
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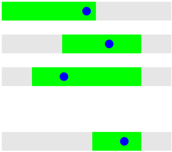

SAT

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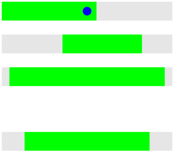

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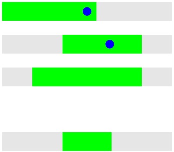

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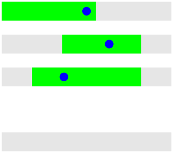

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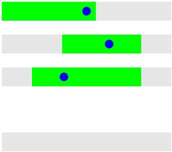

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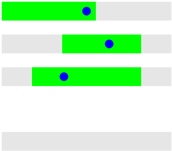

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If at any point the invariant cannot be maintained, it means:

- Some variables $x_1, \ldots, x_n$ have already been assigned values $v_1, \ldots, v_n$ (here $n = i{-}1$): this constitutes a partial model $\mathfrak{M}$;
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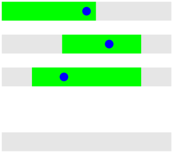

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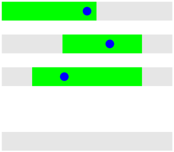

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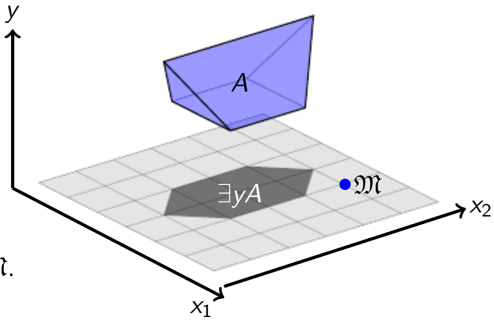
To avoid picking the same values (i.e. the same $\mathfrak{M}$) or another model $\mathfrak{M}'$ that fails "for the same reason" $\mathfrak{M}$ fails, we generalise $\mathfrak{M}$ into a class of failing models and characterise this class by a *conflict explanation*.

Conflict explanation

The conflict explanation is a quantifier-free B (with $\mathrm{fv}(B) \subseteq \{\overrightarrow{x}\}$) over-approximating $\exists y A$:

- $\mathcal{T} \models (\exists y A) \Rightarrow B$
- $\mathfrak{M} \not\models B$

B is an *interpolant* of $\exists y A$ at $\mathfrak{M}$.



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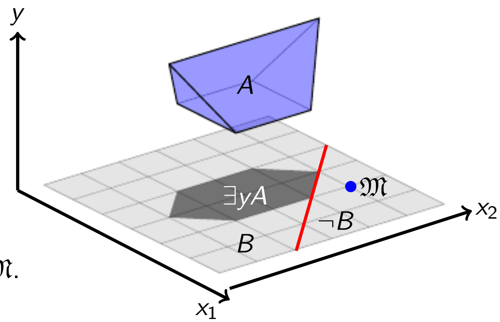

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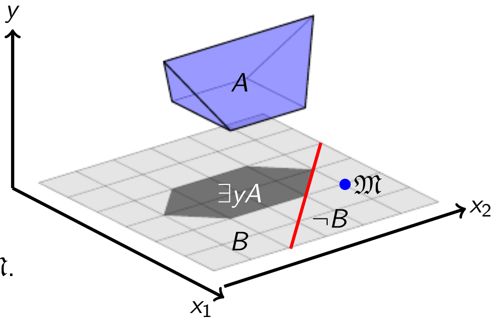

MCSAT considers the theory lemma $A \Rightarrow B$ that rules out not only $\mathfrak{M}$ but a set of similar models (we impose that B be a clause, so $A \Rightarrow B$ is a clause).

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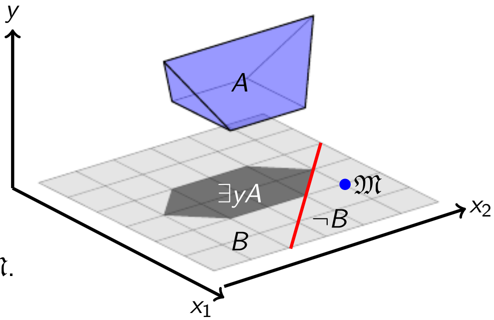

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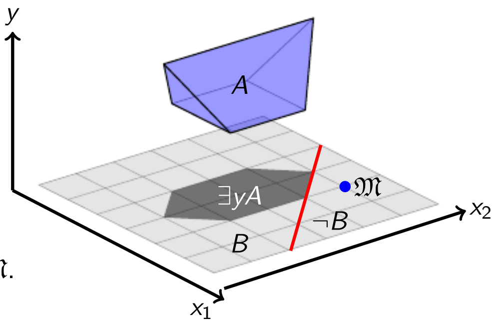

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It backtracks to a point where $A \Rightarrow B$ is no longer violated,
e.g., B no longer evaluates (to false).

MCSAT theories

For a theory $\mathcal{T}$ to be turned into an MCSAT "plugin", we need:

- an efficient way of representing domains of feasible values, and how they are affected (i.e. reduced) by unit constraints;
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  producing interpolants as clauses ,
  satisfying some suitable conditions for termination.

MCSAT framework is implemented in **Yices** (SRI's main SMT-solver), with plugins for Boolean, non-linear arithmetic, EUF (can be mixed), ... and now bitvectors.

2. The bitvector theory in MCSAT

Bitvectors

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Bitvector formulae can be encoded into Boolean logic (one Boolean variable for each bit of each variable): bit blasting.

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On the whole SMTlib bitvector benchmarks,
MCSAT does not perform as well as long established bitblasting solvers (comparison later in this talk),
but there is a decent subset of instances where it performs better. . .

A trivial example

```
(set-info :smt-lib-version 2.6)
(set-logic QF_BV)
(set-info :source |
We verify that (x < y) -> (x + 1 <= y)
...
|)
(set-info :status unsat)
(declare-fun x () (_ BitVec 29980))
(declare-fun y () (_ BitVec 29980))
(assert (bvult x y))
(assert (bvugt (bvadd x (_ bv1 29980)) y))
(check-sat)
(exit)
```

The best 2 solvers of the SMT-comp 2019 (which use bitblasting) cannot solve this.

An MCSAT plugin for bitvectors - encoding domains

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and how they are affected (i.e. reduced) by unit constraints:

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When considered over the bits of a n-bit bitvector variable y, a BDD can encode any set of bitvector values for y.

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Updating the set of feasible values when a constraint becomes unit
corresponds to computing a conjunction of 2 BDDs.

An MCSAT plugin for bitvectors - explanation mechanism

We need an explanation mechanism producing clausal interpolants (satisfying some suitable conditions for termination – easy here);

If $\exists y(C_1 \wedge \cdots \wedge C_m)$ evaluates to false in $\mathfrak{M} = \{x_1 \leftarrow v_1, \ldots, x_n \leftarrow v_n\}$ (i.e., if $v_1, \ldots, v_n$ are the values picked for $x_1, \ldots, x_n$, and $C_1, \ldots, C_m$ are the constraints that leave no feasible values for y)

- **Naive explanation mechanism**: Take
  $\neg B \;=\; x_1 \simeq v_1 \wedge \cdots \wedge x_n \simeq v_n$

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  Bitblast the unsat formula $C_1 \wedge \cdots \wedge C_m \wedge x_1 \simeq v_1 \wedge \cdots \wedge x_n \simeq v_n$,
  and get an *unsat core* identifying the bits of $x_1, \ldots, x_n$ that mattered.

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  Better than the naive mechanism, but still inefficient:
  Many bit-level explanations may be needed to capture a property that could be expressed at the word level.

Word-level explanations

The difficulty is the diversity of word-level bitvector operations.

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To produce an interpolant for $\exists y(C_1 \wedge \cdots \wedge C_m)$ at model $\mathfrak{M}$,

- we get a *conflict core* without redundant constraints, using the QuickXplain algorithm on BDDs; then
- we aggressively rewrite the remaining constraints...

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The fragments:

- **Equality with concat + extract**
- **A fragment of linear bitvector arithmetic**

Equality with concat + extract

$$
\begin{array}{llll}
\text{Constraints} & C & ::= & t \simeq t \mid t \not\simeq t \\
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where e ranges over *evaluable terms*, i.e., terms without variable y (their free variables $x_1, \ldots, x_n$ have values in the current model $\mathfrak{M}$)

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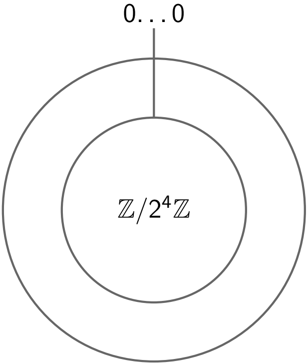
Explanation mechanism given in the paper, utilising *slicing* and model-aware E-graph.

A fragment of bitvector arithmetic - concrete example

$\mathfrak{M} = \{x_1 \leftarrow 1100, x_2 \leftarrow 1101, x_3 \leftarrow 0000\}$

Constraint C_1: $\neg(y \simeq x_1)$
Constraint C_2: $(x_1 \leq^u x_3 + y)$
Constraint C_3: $\neg(y - x_2 \leq^u x_3 + y)$

Space of values for y (feasible ones in white, forbidden ones in red):



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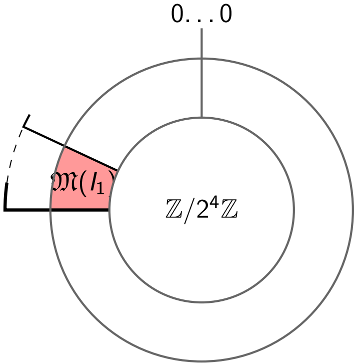

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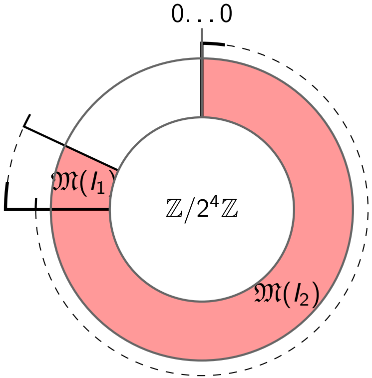

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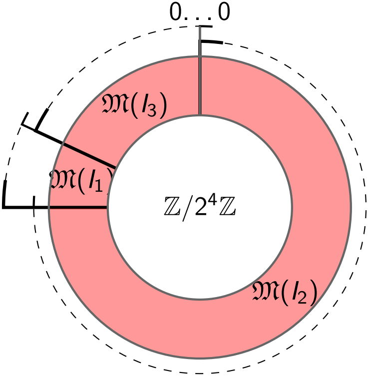

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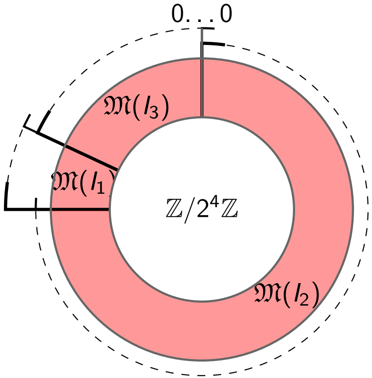
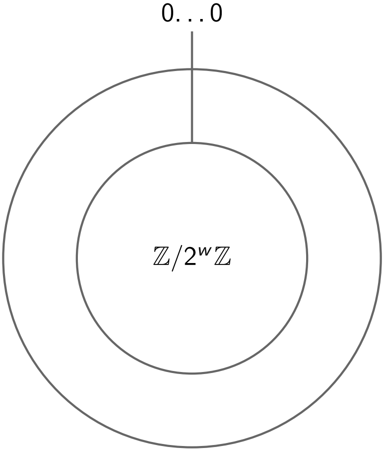


The explanation is $(x_1{+}1) \in I_3 \ \wedge (-x_3) \in I_2 \wedge (x_1 - x_3) \in I_1$

More generally on bitwidth w

Each constraint C_i forbids an interval I_i with interpretation $\mathfrak{M}(I_i)$ and upper bound u_i.



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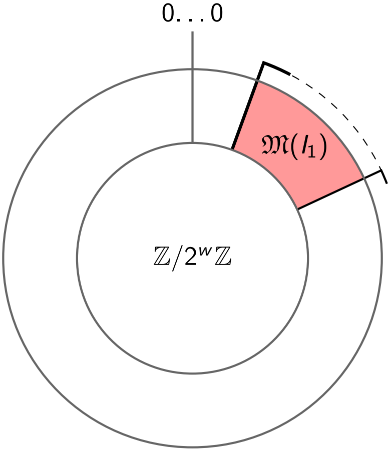

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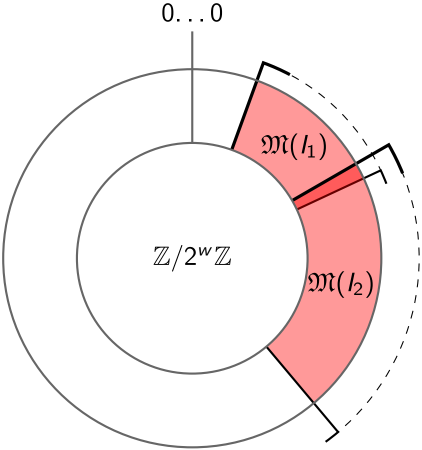

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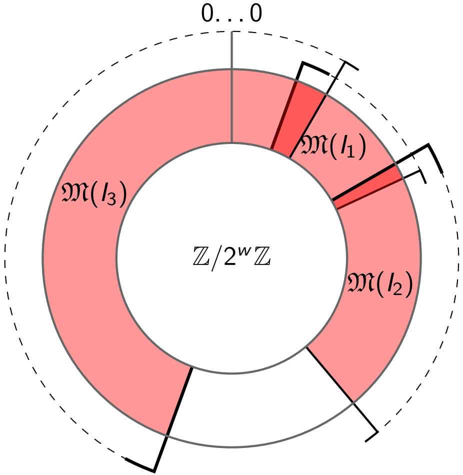

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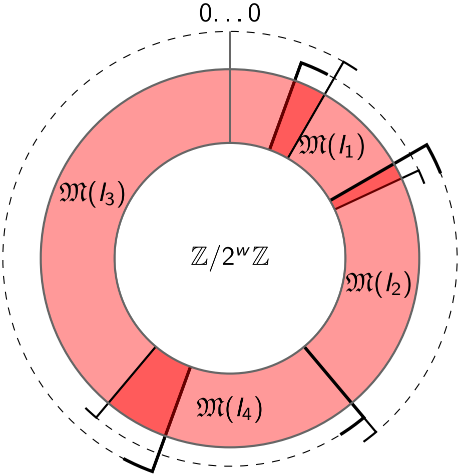

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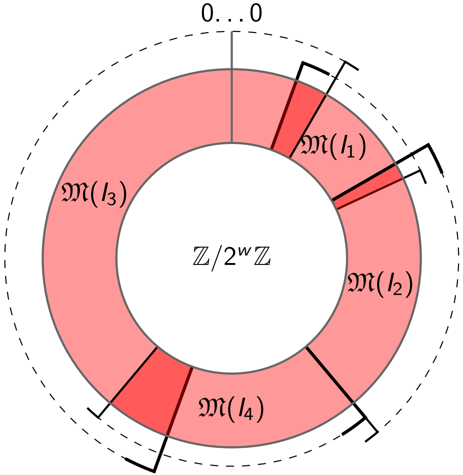

All values in $\mathbb{Z}/2^w\mathbb{Z}$ end up being forbidden because:
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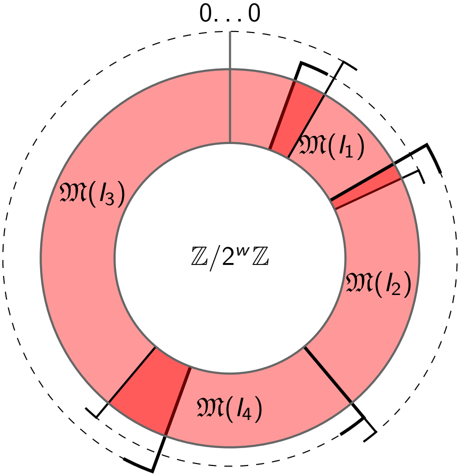

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Things to do in practice

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- Express constraints "$a \in [d; u[$" in the language of linear bv-arithmetic: $a - d <^u u - d$
  In the example, the explanation
  $$(x_1 + 1) \in I_3 \ \wedge (-x_3) \in I_2 \wedge (x_1 - x_3) \in I_1$$
  is expressed as
  $$(x_1 + 1 - x_2 <^u -x_3 - x_2) \wedge (0 <^u x_1) \wedge (-x_3 <^u 1)$$

Extending the method - by enhancing the algorithm

The 12 cases of constraints turning into forbidden intervals capture the following grammar:

$$
\begin{array}{lll}
\text{Constraints} & C & ::= a \;\mid\; \neg a \\
\text{Atoms} & a & ::= e_1 + y \leq^u e_2 + y \;\mid\; e_1 \leq^u e_2 + y \;\mid\; e_1 + y \leq^u e_2
\end{array}
$$

where e_1, e_2 range over *evaluable terms*.

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We can extend the grammar into:

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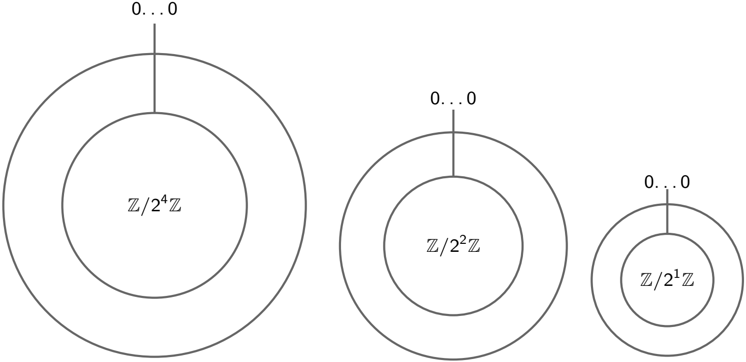

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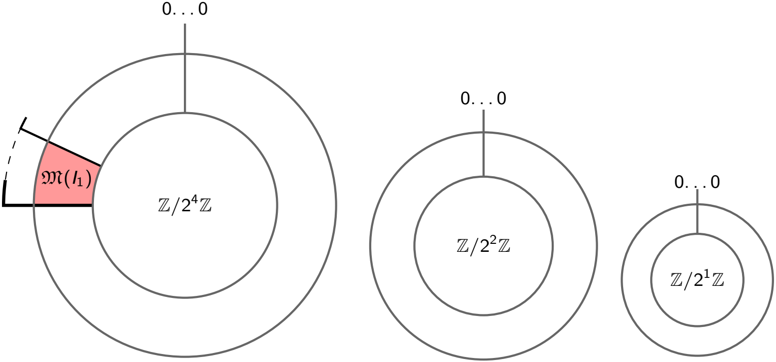

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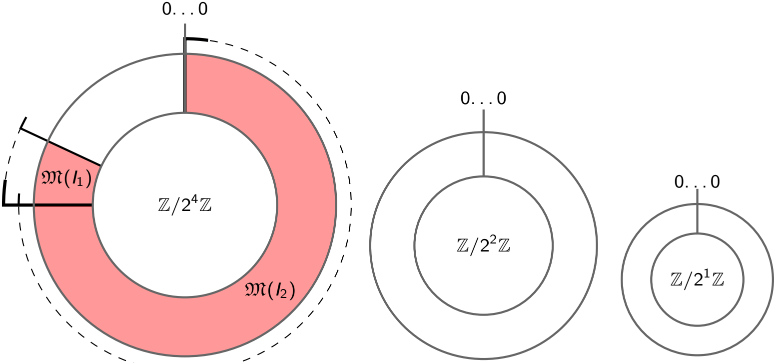

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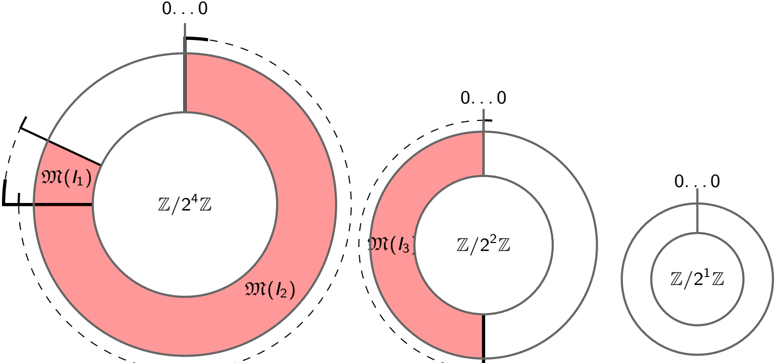

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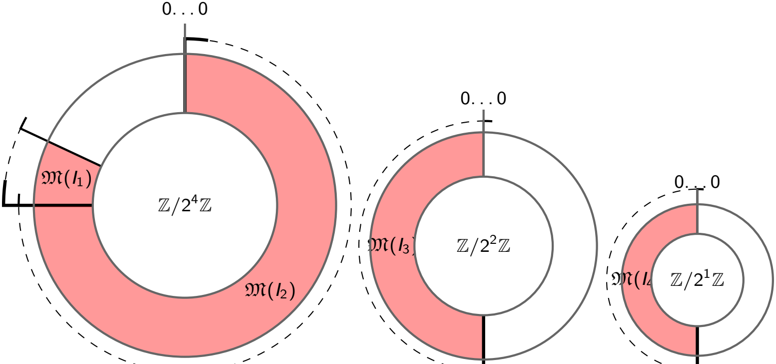

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Terms	t	$::= y[h{:}] \mid t[{:}l] \mid t + e_1 \mid -t \mid 0_k \circ t \mid t \circ 0_k$

where e_1, e_2 range over *evaluable terms*.

Generalization 2:
also with nestings of upper-bit extraction, addition of evaluable terms, negation, and concatenations with 0s (or with evaluable terms).
See Figure 1 in the paper.

Extending the method - by adding rewrites

We normalise the constraints in the conflict with the following rules (Figure 3 in the paper):

$u_1 <^s u_2$	$\rightsquigarrow \neg(u_2 \leq^s u_1)$			$u_1 \leq^s u_2$	$\rightsquigarrow u_1{+}2^{\lvert u_1\rvert-1} \leq^u u_2{+}2^{\lvert u_2\rvert-1}$	
$u_1 <^u u_2$	$\rightsquigarrow \neg(u_2 \leq^u u_1)$			$u_1 \simeq u_2$	$\rightsquigarrow u_1 - u_2 \leq^u 0$	
$u[h{:}l]$	$\rightsquigarrow u[h{:}][{:}l]$			$u[{:}l][h{:}]$	$\rightsquigarrow u[h{+}l{:}][{:}l]$	
$(u_1 \circ u_2)[{:}l]$	$\rightsquigarrow u_1[{:}l{-}\lvert u_2\rvert]$	if $\lvert u_2\rvert \leq l$		$(u_1 \circ u_2)[h{:}]$	$\rightsquigarrow u_2[h{:}]$	if $h \leq \lvert u_2\rvert$
$(u_1 \circ u_2)[{:}l]$	$\rightsquigarrow u_1 \circ u_2[{:}l]$	if not		$(u_1 \circ u_2)[h{:}]$	$\rightsquigarrow u_1[h{-}\lvert u_2\rvert{:}] \circ u_2$	if not
$2^n \times u$	$\rightsquigarrow u[\lvert u\rvert{-}n{:}] \circ 0_n$	$(n < \lvert u\rvert)$		$(u_1{+}u_2)[h{:}]$	$\rightsquigarrow u_1[h{:}] + u_2[h{:}]$	
$\mathrm{bvnot}(u)$	$\rightsquigarrow -(u+1)$			$(u_1 \times u_2)[h{:}]$	$\rightsquigarrow u_1[h{:}] \times u_2[h{:}]$	
$\pm\text{-extend}_k(u)$	$\rightsquigarrow (0_k \circ (u{+}2^{\lvert u\rvert-1})){-}(0_k \circ 2^{\lvert u\rvert-1})$			$(-u)[h{:}]$	$\rightsquigarrow -u[h{:}]$	
$u_1 \circ u_2$	$\rightsquigarrow (u_1 \circ 0_{\lvert u_2\rvert}) + (0_{\lvert u_1\rvert} \circ u_2)$					

Extending the method - by adding rewrites

We normalise the constraints in the conflict with the following rules (Figure 3 in the paper):

$u_1 <^s u_2 \rightsquigarrow \neg(u_2 \leq^s u_1)$			$u_1 \leq^s u_2 \rightsquigarrow u_1{+}2^{\lvert u_1\rvert-1} \leq^u u_2{+}2^{\lvert u_2\rvert-1}$		
$u_1 <^u u_2 \rightsquigarrow \neg(u_2 \leq^u u_1)$			$u_1 \simeq u_2 \rightsquigarrow u_1 - u_2 \leq^u 0$		
$u[h{:}l]$	$\rightsquigarrow u[h{:}][{:}l]$		$u[{:}l][h{:}]$	$\rightsquigarrow u[h{+}l{:}][{:}l]$	
$(u_1{\circ}u_2)[{:}l]$	$\rightsquigarrow u_1[{:}l{-}\lvert u_2\rvert]$	if $\lvert u_2\rvert \leq l$	$(u_1{\circ}u_2)[h{:}]$	$\rightsquigarrow u_2[h{:}]$	if $h \leq \lvert u_2\rvert$
$(u_1{\circ}u_2)[{:}l]$	$\rightsquigarrow u_1 \circ u_2[{:}l]$	if not	$(u_1{\circ}u_2)[h{:}]$	$\rightsquigarrow u_1[h{-}\lvert u_2\rvert{:}] \circ u_2$	if not
$2^n \times u$	$\rightsquigarrow u[\lvert u\rvert{-}n{:}] \circ 0_n$	$(n < \lvert u\rvert)$	$(u_1{+}u_2)[h{:}]$	$\rightsquigarrow u_1[h{:}] + u_2[h{:}]$	
$\mathrm{bvnot}(u)$	$\rightsquigarrow -(u+1)$		$(u_1 \times u_2)[h{:}]$	$\rightsquigarrow u_1[h{:}] \times u_2[h{:}]$	
$\pm\text{-extend}_k(u)$	$\rightsquigarrow (0_k{\circ}(u{+}2^{\lvert u\rvert-1})){-}(0_k{\circ}2^{\lvert u\rvert-1})$		$(-u)[h{:}]$	$\rightsquigarrow -u[h{:}]$	
$u_1{\circ}u_2$	$\rightsquigarrow (u_1{\circ}0_{\lvert u_2\rvert}) + (0_{\lvert u_1\rvert}{\circ}u_2)$				

This allows the plugin to cover (at least) the following grammar:

$$\begin{array}{lll} \text{Atoms} & a ::= & e_1 + t \lessdot e_2 + t \;\mid\; e_1 \lessdot e_2 + t \;\mid\; e_1 + t \lessdot e_2 \;\mid\; e_1 \lessdot e_2 \\ \text{Terms} & t ::= & t[h{:}l] \;\mid\; t + e_1 \;\mid\; -t \;\mid\; e_1 \circ t \;\mid\; t \circ e_1 \;\mid\; \pm\text{-extend}_k(t) \end{array}$$

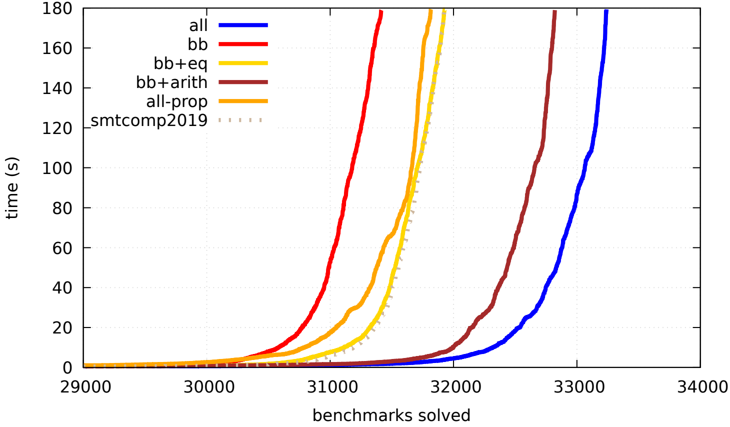
where $\lessdot$ is any comparison symbol in $\{\leq^u, <^u, \leq^s, <^s, \simeq\}$, and terms can also involve arbitrary extracts, sign-extensions, etc

3. Experimentation on the SMTLib benchmarks

Effects of explanation mechanisms and propagation

...on the 41,547 instances in SMTLib (QF_BV)

Timeout is 3 minutes



bb	only the bitblasting baseline
bb+eq	baseline + concat-extract explanation mechanism
bb+arith	baseline + arithmetic explanation mechanism
all	baseline + both mechanisms
all-prop	same as all but with no propagation of bitvector values

Numbers

Total number of instances solved by all: 33,236
(14,174 solved by pure preprocessing + 19,062 using MCSAT)

- 14,313 are solved without ever calling the default bitblasting baseline ($\simeq$ half of the benchmarks are entirely within the two fragments)
- 4,749 instances are solved by a combination of the three explainers.

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MCSAT not as good on the whole, but in the paper we identify classes of instances where MCSAT is better, e.g.,
arithmetic explanation mechanism is insensitive to big bitwidths.
For instance, MCSAT could solve 794 instances for which Boolector+CadiCal timed out.

4. Conclusion

Related work

MCSAT approach to bitvectors first explored in [ZWR16], using

- bitvector intervals and masks to represent domains;
- eager propagation mechanisms instead of interpolation-based conflict-explanations.

Our numbers on SMTLib seem to improve on [ZWR16] quite a bit.

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MCSAT approach to bitvectors first explored in [ZWR16], using

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Our numbers on SMTLib seem to improve on [ZWR16] quite a bit.

Our work extends preliminary work [GLJ17, GLJ19]. Main improvements:

- Use of arbitrary evaluable terms to extend the scopes of the 2 fragments;
- Generalization 2 of the arithmetic explanation mechanism;
- Normalization of conflicts by rewrite rules
- Experimentation

Future work and MCSAT beyond ground SMT-solving

- Extend the fragments little by little, e.g., handling a bigger fragment of bitvector arithmetic, e.g., with arbitrary coefficients for the conflict variable y in polynomials.

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  - *invertibility conditions* [NPR+18]
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  - *invertibility conditions* [NPR+18]
  - other techniques inspired by *quantifier elimination*, e.g., [JC16]

- Even if MCSAT ends up not performing as well as bitblasting on ground instances, it may still be interesting to produce word-level explanations of conflicts.
  Two applications of these MCSAT explanations currently investigated at SRI:
  - *General interpolation problems* in the bitvector theory
  - Solving *quantified problems* in the bitvector theory

Questions?

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