Proofs in Conflict-Driven Theory Combination

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and Natarajan Shankar

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Context: Satisfiability Modulo Theories (SMT)

CDCL (Conflict-Driven Clause Learning)

- procedure for deciding the satisfiability of Boolean formulae
- uses assignments of Boolean values to variables, e.g., $l \leftarrow \mathsf{true}$

MCSAT (Model-Constructing Satisfiability) [dMJ13, Jov17]

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MCSAT and CDSAT can explicitly provide, for satisfiable formulae, the model's assignments of values to variables

This paper concerns the dual situation of unsatisfiable formulae: there exists a proof (of the formula's negation)

Questions addressed

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Here, we start by adding learning mechanisms to CDSAT.

Conflict-driven theory combination

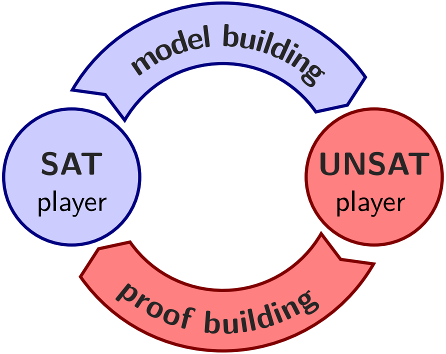
The CDSAT system - with learning

Proof production

1. Conflict-driven theory combination

Conflict-driven reasoning

2-player game to determine whether a formula is satisfiable.
It involves a trail where a putative model is being specified.
It relies on a notion of conflict between the putative model and the formula it should satisfy.



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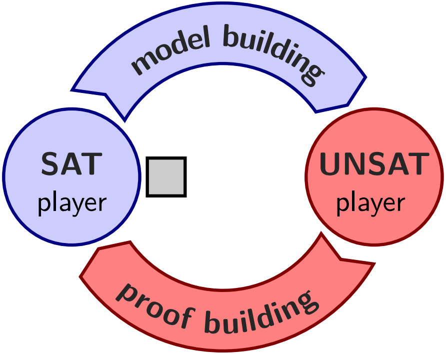

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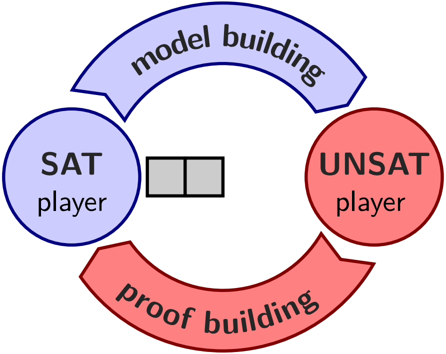

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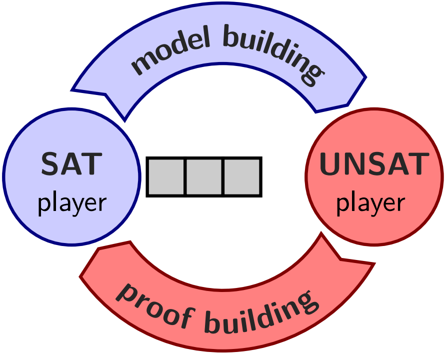

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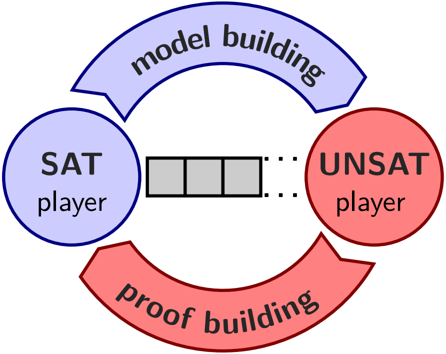

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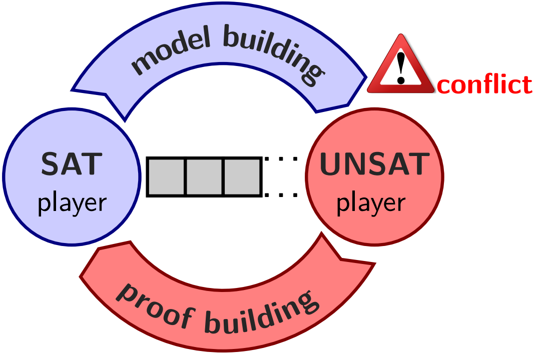

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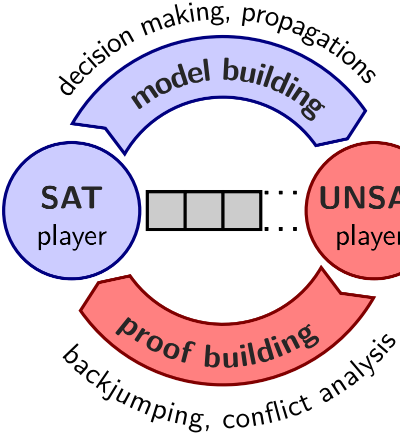

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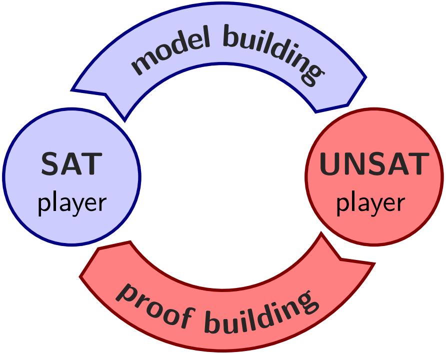

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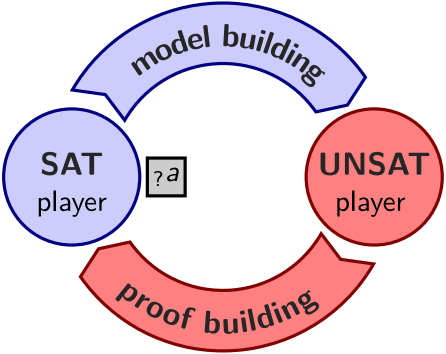

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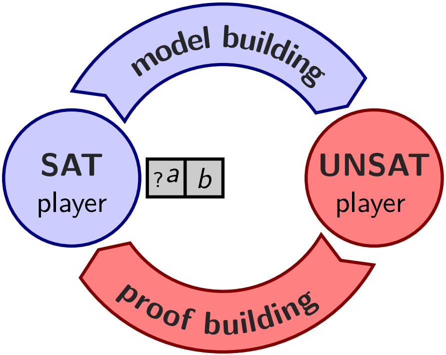

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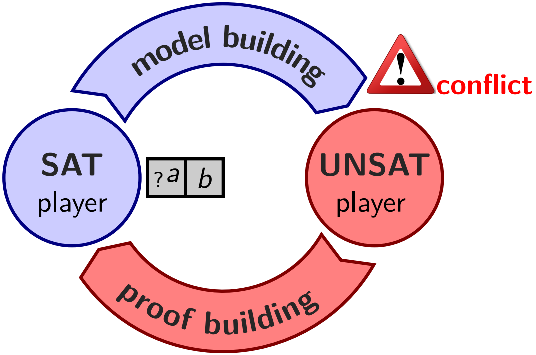

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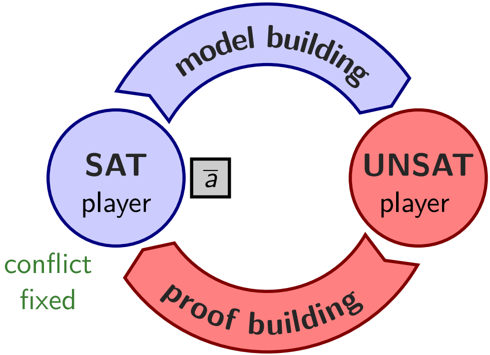

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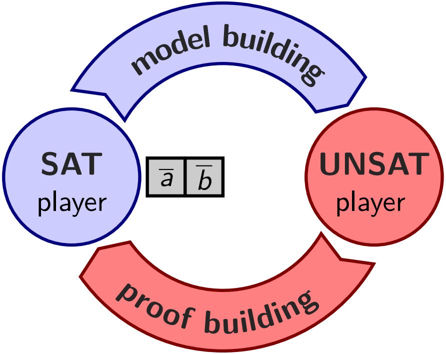

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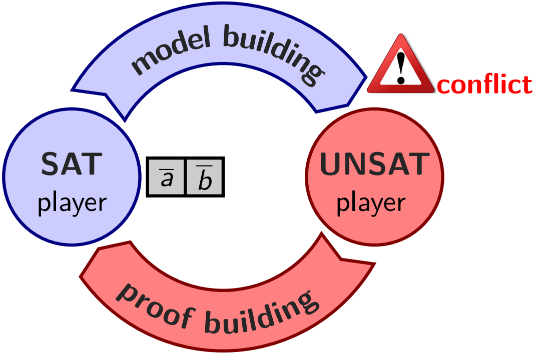

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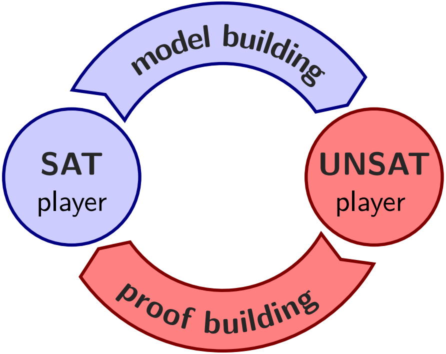

Conflict-driven reasoning can be used for (other) theories

$$\overbrace{(-2\cdot x - y < 0)}^{l_0}, \qquad \overbrace{(x + y < 0)}^{l_1}, \qquad \overbrace{(x < -1)}^{l_2}$$

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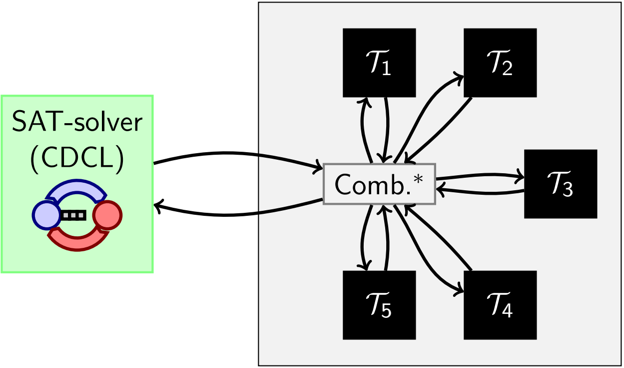
- and so on...
  (when there is no guess to undo, problem is UNSAT)

Traditional architecture of SMT-solving



* e.g. equality sharing / Nelson-Oppen [NO79]

In CDSAT

. . . the theory combination is organised directly in the main conflict-driven loop:

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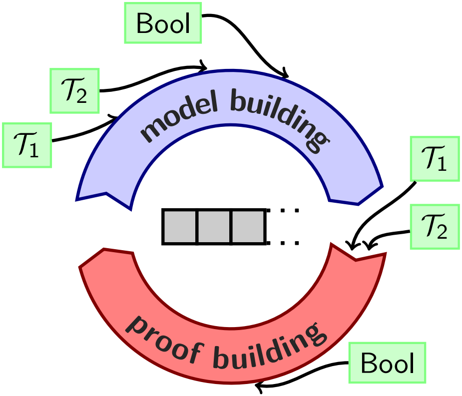

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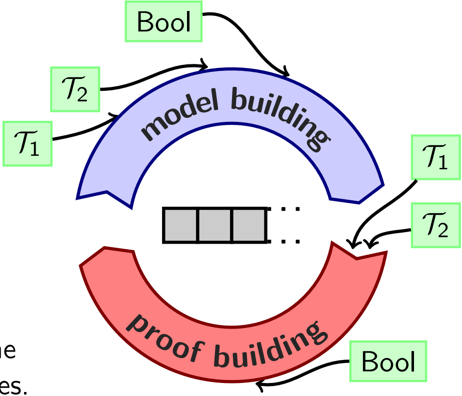

Features of conflict-driven satisfiability:

- Boolean theory can have the same status as other theories.
- Theory-specific reasoning often consists of fine-grained reasoning inferences, e.g., Fourier-Motzkin resolution for LRA:
  $$(t_1 < x), (x < t_2) \vdash t_1 < t_2$$

2. The CDSAT system - with learning

What is a theory module?

A set of inferences of the form

$$(t_1 \leftarrow \mathfrak{c}_1), \ldots, (t_k \leftarrow \mathfrak{c}_k) \vdash_{\mathcal{T}} (l \leftarrow \mathfrak{b})$$

where

- each $t_i \leftarrow \mathfrak{c}_i$ is a single $\mathcal{T}$-assignment
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- Soundness requirement:
  Every model of the premisses is a model of the conclusion:
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Examples:

$(x \leftarrow \sqrt{2}), (y \leftarrow \sqrt{2}) \vdash_{\mathrm{NLRA}} (x \cdot y \simeq 2)$ (evaluation inference)

$(l_1 \vee \cdots \vee l_n), \overline{l_1} \ldots, \overline{l_{n-1}} \vdash_{\mathrm{Bool}} l_n$ (unit propagation)

What is a theory module? (Equality inferences)

All theory modules have the equality inferences:

$$
\begin{array}{rll}
t_1 \leftarrow \mathfrak{c}_1, t_2 \leftarrow \mathfrak{c}_2 \vdash_{\mathcal{T}} & t_1 \simeq t_2 & \text{if } \mathfrak{c}_1 \text{ and } \mathfrak{c}_2 \text{ are the same value} \\
t_1 \leftarrow \mathfrak{c}_1, t_2 \leftarrow \mathfrak{c}_2 \vdash_{\mathcal{T}} & t_1 \not\simeq t_2 & \text{if } \mathfrak{c}_1 \text{ and } \mathfrak{c}_2 \text{ are distinct values} \\
\\
\vdash_{\mathcal{T}} & t_1 \simeq t_1 & \text{reflexivity} \\
t_1 \simeq t_2 \vdash_{\mathcal{T}} & t_2 \simeq t_1 & \text{symmetry} \\
t_1 \simeq t_2, t_2 \simeq t_3 \vdash_{\mathcal{T}} & t_1 \simeq t_3 & \text{transitivity}
\end{array}
$$

CDSAT states

Search states: simply trails.

A trail is a stack of justified assignments ${}_{H\vdash}(t{\leftarrow}\mathfrak{c})$ and decisions ${}_{?}(t{\leftarrow}\mathfrak{c})$ coming from different theories

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Example (trail grows from left to right):

$$ {}_{\emptyset\vdash}(x \simeq z), {}_{\emptyset\vdash}(y \simeq z), {}_{?}(x{\leftarrow}\sqrt{2}), {}_{?}(y{\leftarrow}\mathrm{blue}), {}_{?}(x{\leftarrow}\mathrm{red}), {}_{H\vdash}(x \neq y) $$

where H is $\{(y{\leftarrow}\mathrm{blue}),\ (x{\leftarrow}\mathrm{red})\}$

Everything is on the trail, including assertions from the input problem, with empty justifications

(e.g., ${}_{\emptyset\vdash}(C{\leftarrow}\mathrm{true})$ for an input clause C),

CDSAT states

Search states: simply trails.
A trail is a stack of justified assignments ${}_{H\vdash}(t{\leftarrow}\mathfrak{c})$ and decisions ${}_{?}(t{\leftarrow}\mathfrak{c})$ coming from different theories
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In this paper, new rule for solving/exiting conflicts: Learn

Example: exiting a conflict without learning a clause

Input problem H_0 including: $(\neg l_2 \vee \neg l_4 \vee \neg l_5)$

with $l_4 = (x \leq y)$ and $l_5 = (f(x) \leq f(y))$ in a theory where f is monotonic

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First conflict: $\langle \Gamma; (\neg l_2 \vee \neg l_4 \vee \neg l_5), l_2, l_4, l_5 \rangle$

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Resolving l_5: $\langle \Gamma; (\neg l_2 \vee \neg l_4 \vee \neg l_5), l_2, l_4 \rangle$

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In first conflict, both l_4 and l_5 depend on the latest decision ${}_{?}l_4$.
After applying Resolve, only l_4 does. Time to stop conflict analysis.

Example: exiting a conflict without learning a clause

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Rule Learn can exit the conflict with trail
$\Gamma_0,\ {}_{?}A_1,\ {}_{?}l_2,\ {}_{H\vdash}\overline{l_4}$

where H is $\{(\neg l_2 \vee \neg l_4 \vee \neg l_5), l_2\}$

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Initial trail Γ_0 including: ${}_{\emptyset\vdash}(\neg l_2 \vee \neg l_4 \vee \neg l_5)$

Search rules extend Γ_0 into $\Gamma \; = \; \Gamma_0,\ {}_{?}A_1,\ {}_{?}l_2,\ {}_{?}A_3,\ {}_{?}l_4,\ {}_{l_4\vdash}l_5$
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Example: exiting a conflict learning a clause

Input problem H_0 including: $(\neg l_2 \vee \neg l_4 \vee \neg l_5)$
with $l_4 = (x \leq y)$ and $l_5 = (f(x) \leq f(y))$ in a theory where f is monotonic
Initial trail Γ_0 including: ${}_{\emptyset\vdash}(\neg l_2 \vee \neg l_4 \vee \neg l_5)$

Search rules extend Γ_0 into $\Gamma \;=\; \Gamma_0,\ {}_{?}A_1,\ {}_{?}l_2,\ {}_{?}A_3,\ {}_{?}l_4,\ {}_{l_4\vdash}l_5$
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Rule Learn can exit the conflict and learn a clause:
$\Gamma_0,\ {}_{?}A_1,\ {}_{?}l_2,\ {}_{H'\vdash}(\neg l_2 \vee \neg l_4)$
where H' is $\{(\neg l_2 \vee \neg l_4 \vee \neg l_5)\}$

Example: exiting a conflict learning a clause

Input problem H_0 including: $(\neg l_2 \vee \neg l_4 \vee \neg l_5)$
with $l_4 = (x \leq y)$ and $l_5 = (f(x) \leq f(y))$ in a theory where f is monotonic
Initial trail Γ_0 including: ${}_{\emptyset}\vdash(\neg l_2 \vee \neg l_4 \vee \neg l_5)$

Search rules extend Γ_0 into $\Gamma \ = \ \Gamma_0,\ {}_{?}A_1,\ {}_{?}l_2,\ {}_{?}A_3,\ {}_{?}l_4,\ {}_{l_4}\vdash l_5$
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where H' is $\{(\neg l_2 \vee \neg l_4 \vee \neg l_5)\}$
Then Deduce can derive $\overline{l_4}$ as before:
$\Gamma_0,\ {}_{?}A_1,\ {}_{?}l_2,\ {}_{H'}\vdash(\neg l_2 \vee \neg l_4),\ {}_{\{(\neg l_2 \vee \neg l_4), l_2\}}\vdash \overline{l_4}$

Example: exiting a conflict learning a clause & restarting

Input problem H_0 including: $(\neg l_2 \vee \neg l_4 \vee \neg l_5)$
with $l_4 = (x \leq y)$ and $l_5 = (f(x) \leq f(y))$ in a theory where f is monotonic
Initial trail Γ_0 including: ${}_{\emptyset}\!\vdash\!(\neg l_2 \vee \neg l_4 \vee \neg l_5)$

Search rules extend Γ_0 into $\Gamma\ =$ $\Gamma_0,\ {}_{?}A_1,\ {}_{?}l_2,\ {}_{?}A_3,\ {}_{?}l_4,\ {}_{l_4}\!\vdash\! l_5$
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First conflict: $\langle \Gamma; (\neg l_2 \vee \neg l_4 \vee \neg l_5), l_2, l_4, l_5 \rangle$
Resolving l_5: $\langle \Gamma; (\neg l_2 \vee \neg l_4 \vee \neg l_5), l_2, l_4 \rangle$
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After applying Resolve, only l_4 does. Time to stop conflict analysis.

Rule Learn can exit the conflict and learn a clause, and restart:
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where H' is $\{(\neg l_2 \vee \neg l_4 \vee \neg l_5)\}$

The Learn rule introduced in this paper

$$\langle \Gamma; E \uplus H \rangle \longrightarrow \Gamma', {}_{E}\vdash L \quad \text{if } L \text{ is a "clausal form of } H\text{", } L \notin \Gamma,\ \overline{L} \notin \Gamma$$

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"Clausal forms of H" **reify** H in Boolean logic:

$$((\bigwedge_{(l \leftarrow \mathsf{true}) \in H} l) \wedge (\bigwedge_{(l \leftarrow \mathsf{false}) \in H} \neg l)) \leftarrow \mathsf{false}$$

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This rule

- generalises the CADE'2017 one (sufficient for completeness)
- models clause learning by reifying (Boolean parts of) conflicts
- models clause learning + restarts,
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Which version to apply depends on your search strategy (particularly for restarts)
All version are OK with respect to termination of CDSAT

3. Proof production

Soundness invariants, and rules that may affect them

- For every assignment ${}_{H\vdash}A$ on the trail, $H \models A$;
- For every conflict state $\langle \Gamma; E \rangle$, $E \models \bot$.

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Let $\mathcal{T}$ be a theory with a specific $\mathcal{T}$-module.

Deduce

$$\Gamma \quad \longrightarrow \quad \Gamma, {}_{J\vdash}(t \leftarrow \mathfrak{b}) \qquad \text{if } J \vdash_{\mathcal{T}} (t \leftarrow \mathfrak{b}) \text{ and } J \subseteq \Gamma, \text{ and } t \leftarrow \overline{\mathfrak{b}} \text{ is not in } \Gamma$$

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Conflict

$$\Gamma \longrightarrow \langle \Gamma; J, (t{\leftarrow}\overline{\mathfrak{b}}) \rangle \quad \text{if } J \vdash_{\mathcal{T}} (t{\leftarrow}\mathfrak{b}) \text{ and } J \subseteq \Gamma, \text{ and } t{\leftarrow}\overline{\mathfrak{b}} \text{ is in } \Gamma$$

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Resolve

$$\langle \Gamma; E \uplus \{A\} \rangle \longrightarrow \langle \Gamma; E \cup H \rangle \quad \text{if } {}_{H\vdash}A \text{ is in } \Gamma$$

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Deduce

$\Gamma \longrightarrow \Gamma, {}_{J\vdash}(t \leftarrow \mathfrak{b})$ if $J \vdash_{\mathcal{T}} (t \leftarrow \mathfrak{b})$ and $J \subseteq \Gamma$, and $t \leftarrow \overline{\mathfrak{b}}$ is not in Γ

Conflict

$\Gamma \longrightarrow \langle \Gamma; J, (t \leftarrow \overline{\mathfrak{b}}) \rangle$ if $J \vdash_{\mathcal{T}} (t \leftarrow \mathfrak{b})$ and $J \subseteq \Gamma$, and $t \leftarrow \overline{\mathfrak{b}}$ is in Γ

Resolve

$\langle \Gamma; E \uplus \{A\} \rangle \longrightarrow \langle \Gamma; E \cup H \rangle$ if ${}_{H\vdash}A$ is in Γ

Learn

$\langle \Gamma; E \uplus H \rangle \longrightarrow \Gamma', {}_{E\vdash}L$ if L is a "clausal form" of H $L \notin \Gamma$, $\overline{L} \notin \Gamma$, and $E \subseteq \Gamma'$

Theory proofs

To keep track of the soundness invariants,
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Each theory module comes with a "proof annotation system"

$$(t_1 \leftarrow \mathfrak{c}_1), \ldots, (t_k \leftarrow \mathfrak{c}_k) \vdash_{\mathcal{T}} (l \leftarrow \mathfrak{b})$$

is annotated as

$${}^{a_1}(t_1 \leftarrow \mathfrak{c}_1), \ldots, {}^{a_k}(t_k \leftarrow \mathfrak{c}_k) \vdash_{\mathcal{T}} j_{\mathcal{T}} : (l \leftarrow \mathfrak{b})$$

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Examples:

$^{a_1}(x \leftarrow \sqrt{2}), {}^{a_2}(y \leftarrow \sqrt{2}) \vdash_{\mathrm{NLRA}} \mathrm{eval}(\{a_1, a_2\}) : (x \cdot y \simeq 2)$
(evaluation inference)

$^{a_0}(l_1 \vee \cdots \vee l_n), {}^{a_1}(\overline{l_1}), \ldots, {}^{a_{k-1}}(\overline{l_{n-1}}) \vdash_{\mathrm{Bool}} \mathrm{UP}(a_0, \{a_1, \ldots, a_n\}) : l_n$
(unit propagation)

Proof-terms and proof-carrying CDSAT

- A proof-carrying trail is a stack
  - of justified assignments ${}_{H\vdash j:}(t\leftarrow\mathfrak{c})$
  - and decisions ${}_{?}(t\leftarrow\mathfrak{c})$
- A proof-carrying conflict state is of the form $\langle\Gamma;\ H\ ;\ c\rangle$

...where j and c respectively range over

$$\begin{array}{lrl} \text{Deduction proof terms} & j ::= & \mathsf{in} \ \mid\ j_{\mathcal{T}} \ \mid\ \mathsf{lem}(H.c) \\ \text{Conflict proof term} & c ::= & \mathsf{cfl}(j_{\mathcal{T}}, a) \ \mid\ \mathsf{res}(j, {}^{a}A.c) \end{array}$$

in	annotates an input assignment,
$j_{\mathcal{T}}$	ranges over theory proofs for $\mathcal{T}$, used for Deduce
$\mathsf{lem}(H.c)$	annotates justified assignments that Learn places on trail (clausal forms of H), binding the identifiers of H in c
$\mathsf{cfl}(j_{\mathcal{T}}, a)$	annotates a conflict when it is created by Conflict
$\mathsf{res}(j, {}^{a}A.c)$	annotates a conflict resulting from the Resolve rule, binding a in c

Provability invariants that proof-terms keep track of

$$\frac{A \text{ is an input}}{\emptyset \vdash \mathsf{in} : A} \qquad \frac{J \vdash_{\mathcal{T}} j_{\mathcal{T}} : L}{J \vdash j_{\mathcal{T}} : L} \qquad \frac{E \uplus H \vdash c : \bot}{E \vdash \mathsf{lem}(H.c) : L}\ L \text{ clausal form of } H$$

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Rules of CDSAT are adapted so as to use those proof-terms, and the soundness invariants are materialised as:

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The proof system above can be seen as glueing a collection of inference systems $(\vdash_{\mathcal{T}})_{\mathcal{T}}$
CDSAT is a search procedure for the resulting system

Satisfiability Modulo Assignments (SMA)

An SMT-problem with input clauses $C_1, \ldots, C_n$ is treated by running CDSAT on the initial trail ${}_{\emptyset\vdash\mathsf{in}:} C_1, \ldots, {}_{\emptyset\vdash\mathsf{in}:} C_n$

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But the CDSAT system can accept inputs with first-order assignments, e.g: ${}_{\emptyset\vdash\mathsf{in}:}(x\leftarrow{}^3/_4)$, ${}_{\emptyset\vdash\mathsf{in}:}(x\leq y)$, ${}_{\emptyset\vdash\mathsf{in}:}(y\leq 0)$
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Theory modules do not have to provide theory proofs $H \vdash_{\mathcal{T}} j_{\mathcal{T}} : L$
if H contains a first-order assign. (typically: evaluation inferences)

Different views about proof objects

Proof-carrying CDSAT can be considered exactly as defined above, where $\mathsf{in}, j_{\mathcal{T}}, \mathsf{lem}(H.c), \mathsf{cfl}(j_{\mathcal{T}}, a), \mathsf{res}(j, {}^{a}A.c)$ are terms.

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Just interpret the terms in that format after the run

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Alternatively,
proof-carrying CDSAT can directly manipulate proofs in the format,
if equipped with the operations corresponding to the term constructs.
The proof-terms *denote* the manipulated proofs,
but are never constructed.

Example: resolution proofs

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If input does contains first-order assignments (SMA problems)
the resolution format has to be slightly extended,
so that it manipulates guarded clauses of the form

$$\{(t_1 \leftarrow \mathfrak{c}_1), \ldots, (t_n \leftarrow \mathfrak{c}_n)\} \Rightarrow C$$

where $(t_1 \leftarrow \mathfrak{c}_1), \ldots, (t_n \leftarrow \mathfrak{c}_n)$ are first-order assign. guarding clause C
Details in the paper.

LCF: answers that are correct-by-construction

Other “proof format”:

- A deduction proof j with $H \vdash j : L$ is the pair $\langle H, L \rangle$, and
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No proof object needs to be built in memory

CDSAT is well-suited to the LCF approach 1/2

Given a type `assign` for multiple assignments
and `single_assign` for singleton assignments,
a trusted kernel defines

```
type deduction = assign*single_assign
type conflict  = assign
```

and exports

```
type deduction
type conflict
in    : single_assign -> deduction
coerc : 'k theory_handler
        -> 'k theory_proof -> deduction
lem   : conflict -> assign -> deduction
cfl   : 'k theory_handler
          -> 'k theory_proof -> conflict
res   : deduction -> conflict -> conflict
```

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If the empty assignment is constructed in type `conflict`, input problem is guaranteed to be unsat, provided the kernel primitives and the implementation of theory proofs are trusted (code for the search plan does not have to be certified)

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Answer is correct-by-construction, no proof object in memory.

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    If inputs contain first-order assignments,
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  - Proof-terms can be convenient for translations to proof assistants (c.f. SMTCoq [AFG+11])
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Ongoing and future work

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- Issue of cost: Penalty of building proof-terms?
  Penalty of having a code developed in the correct-by-construction approach?
- Use proof-terms for interpolation?
  (See Tanja Schindler's talk on interpolation in a related context!
  16:30 at VMCAI)

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Adapting the rules

Deduce

$\Gamma \longrightarrow \Gamma, {}_{J \vdash j_{\mathcal{T}}:}(t \leftarrow \mathfrak{b})$ if $J \vdash_{\mathcal{T}} j_{\mathcal{T}} : (t \leftarrow \mathfrak{b})$, $J \subseteq \Gamma$, and $t \leftarrow \overline{\mathfrak{b}}$ is not in Γ

Conflict

$\Gamma \longrightarrow \langle \Gamma;\ J, (t \leftarrow \overline{\mathfrak{b}})\ ;\ \mathrm{cfl}(j_k, a) \rangle$ if $J \vdash_{\mathcal{T}} j_{\mathcal{T}} : (t \leftarrow \mathfrak{b})$, $J \subseteq \Gamma$, and $t \leftarrow \overline{\mathfrak{b}}$ is in Γ with id a

Resolve

$\langle \Gamma;\ E \uplus \{A\}\ ;\ c \rangle \longrightarrow \langle \Gamma;\ E \cup H\ ;\ \mathrm{res}(j, {}^{a}A.c) \rangle$

if ${}_{H \vdash j:} A$ is in Γ with id a

Learn

$\langle \Gamma;\ E \uplus H\ ;\ c \rangle \quad \longrightarrow \quad \Gamma', {}_{E \vdash \mathrm{lem}(H.c):} L$ if L is a "clausal form" of H, $L \notin \Gamma$, $\overline{L} \notin \Gamma$, and $E \subseteq \Gamma'$