Mechanically Assisted Examination of Begging the Question in Anselm's Ontological Argument

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Begging the Question: Informal Usage

- Often taken to mean "to invite the question"
- E.g., "The Brexit result begs the question 'why do people vote against their self interest?' "
- Correct usage is whatever native speakers say
- But. . .

Begging the Question: Formal Usage

- In logic and argumentation it means
  - Assuming that which is to be proved

  i.e., a form of circular meaning

- Comes from medieval translations of Aristotle
  - Beg: "to take for granted without warrant" [OED]

Begging the Question: in Argumentation

- Traditionally discussed in context of informal or semi-formal argumentation and dialectics
- One of the premises is equivalent to the conclusion
- Or restates it in different words
- Some consider it a fallacy
- Others say valid but unpersuasive
- May still be interesting

Begging the Question: in Formal Logic

- Deductive proofs do not generate new knowledge
- Conclusion is always implicit in the premises
- But can generate surprise or insight
- My criterion for question begging is
  - The conclusion or proof is represented so directly in the premises as to vitiate hope of surprise or insight
- I'll introduce 3 interpretations: strict, weak, indirect begging
- And will examine first- and higher-order versions of Anselm's Ontological Argument for these kinds of question begging
  - I've also examined modal versions (another paper), see later

Begging the Question: Role of Other Premises

- In informal treatments, the question begging premise is equivalent to the conclusion, on its own
- But if that is so, what are the other premises for?
- I think criteria for whether a premise begs the question should apply after we have accepted the other premises

Begging the Question: Strict Case

- Conclusion C
- Questionable premise (may be a conjunction) Q
- Other premises P
- Can do the proof: $P, Q \vdash C$
- But actually $P \vdash Q = C$
  - i.e., Q is equivalent to C, given P
- So can also prove Q from C: $P, C \vdash Q$

Mechanization

- Detecting and demonstating question begging requires exploring variants of a deductive proof
- Tedious and error prone by hand
- Mechanization makes it fast and inexpensive, and reliable
- My goal: show the utility of Verification Systems in doing this
- These are tools from Computer Science, generally used for analysis of algorithms and software or hardware designs
- Comprise a specification language
  - A rich, usually higher-order, logic
- And a collection of powerful deductive engines
  - e.g., satisfiability solvers for combinations of theories, model checkers, automated & interactive theorem provers
- I'll use PVS, available since 1993, 3,000 citations, CAV Award

Application: Anselm's Ontological Argument

- I assume most here are familiar with the Ontological Argument
- Proof of the existence of God
- Due to St. Anselm (Proslogion Chapter II, 1079)
- Modern rendition, alternatives in braces:
  1. We can conceive of {that/something} than which there is no greater
  2. If that thing does not exist in reality, then we can conceive of a greater thing—namely, something {just like it} that does exist in reality
  3. Thus, either the greatest thing exists in reality or it is not the greatest thing
  4. Therefore the greatest thing exists in reality
  5. (That's God)
- I'll start with Oppenheimer and Zalta's rendition

Oppenheimer and Zalta's Rendition in PVS

```
oandz: THEORY
BEGIN
  beings: TYPE
  x, y: VAR beings

  >: (trichotomous?[beings])          % Predicate Subtype

  God?(x): bool = NOT EXISTS y: y > x
  re?(x): bool                        % exists in reality

  ExUnd: AXIOM EXISTS x: God?(x)
  Greater1: AXIOM FORALL x: (NOT re?(x) => EXISTS y: y > x)

  God_re: THEOREM re?(the(God?))      % definite description
END oandz
```

Aside: Definitions from the PVS Prelude

```
orders [T: TYPE]: THEORY
x, y: VAR T
< : VAR pred[[T, T]]
trichotomous?(<): bool = (FORALL x, y: x < y OR y < x OR x = y)


....
a: VAR setof[T]
the(p: (singleton?)): (p)
singleton?(a): bool = (EXISTS (x:(a)): (FORALL (y:(a)): x = y))
...
x: VAR T
choose(p: (nonempty?)): (p)
nonempty?(a): bool = NOT empty?(a)
empty?(a): bool = (FORALL x: NOT member(x, a))
member(x, a): bool = a(x)
```

Analysis of Oppenheimer and Zalta's Rendition

- PVS generates a proof obligation (TCC) to ensure definite description is well-defined (i.e., exists and is unique)
  - Proof of that uses `ExUnd` and trichotomy of `>`
- PVS easily proves `God_re` from `Greater1`
- And proves `Greater1` from `God_re`, i.e., circularity!
  - Also needs trichotomy of `>` to do that
- And hence that `Greater1` from `God_re` are equivalent
- Thus `Greater1` strictly begs the question
- Already noted by Paweł Garbacz

Eder and Ramharter's First Rendition

- O&Z use a definite description: that than which no greater
- Formalized as `the(God?)`
- Need trichotomy of `>` to ensure this is well-defined
  (exists and is unique)
- Eder and Ramharter say this is an incorrect reading,
  should be: something than which no greater
- Can then eliminate trichotomy
- Conclusion becomes

```
God_re_alt:  THEOREM EXISTS x:  God?(x) and re?(x)
```

- `Greater1` no longer begs the question
- But `>` is now unconstrained
- Could be the empty relation

Analysis of Eder and Ramharter's Rendition

- In PVS, we can exhibit a model of E&R's rendition

```
eandr1interp: THEORY
BEGIN
 IMPORTING eandr1{{   % exhibiting a model of E&R rendition
  beings := nat,
  > := LAMBDA (x, y: nat): FALSE,
  re? := LAMBDA (x: nat): TRUE}}
AS model
END eandr1interp
```

- In the model, `beings` become natural numbers, `>` is empty (nothing is greater than anything else) and `re?` is everywhere true (everything exists in reality)
- PVS generates proof obligations to ensure `AXIOMs` of the interpreted theory are theorems in the model
- For `ExUnd`, we exhibit `42` as satisfying `God?`

Continuing Analysis of Eder and Ramharter's Rendition

- Such a model seems contrary to the intent of the Argument
- Surely it is not intended that something than which there is no greater is so because nothing is greater than anything else
- Should require some minimal constraint on `>` to eliminate such vacuous models
- Plausible constraint is that `>` be trichotomous
- But then `Greater1` again begs the question
- A weaker condition is that only beings satisfying the `God?` predicate are required to stand in the `>` relation to others

```
FORALL x,y:  God?(x) => x>y or x=y
```

- But then again `Greater1` begs the question

Begging the Question: Weak Case

- Questionable premise does not strictly beg the question
- But does so when other premises are lightly augmented
- We have: $P, Q \vdash C$
- But $P, C \nvdash Q$
- However, can find P_2 such that
- But $P, P_2, C \vdash Q$
- And then obviously $P, P_2 \vdash Q = C$
- Say that Q weakly begs the question under augmentation P_2
- Significance depends on how "small" and "natural" is P_2
- But. . .
- Can evade detection by making Q more general than needed
- For example. . .

Eder and Ramharter's Second Rendition

- Eder and Ramharter consider `Greater1` unsatisfactory because it does not express "conceptions presupposed by the author"
- Says nothing about what it means to be greater other than the contrived connection to exists in reality
- They propose alternative premise Greater2:

```
FORALL x, y:  (re?(x) AND NOT re?(y) => x > y)
```

- Also need to add another premise

```
Ex_re:  AXIOM EXISTS x:  re?(x)
```

- `Greater2` is not strictly begging
- However, `Greater2` and `Ex_re` together entail `Greater1`
- So it looks suspicious
- Could solve for a P_2 to show that it weakly begs
- But difficult and P_2 may not be small and natural
- Is there some other way to indict `Greater2`?

Indirectly Begging the Question

- We start a PVS proof of `God_re_alt`, introduce premises `ExUnd` and `Ex_re`, expand definition of `God?`, perform routine steps of Skolemization, instantiation, and propositional simplification
- And we arrive at the following sequent

```
[-1]  re?(x!1)   % Terms such as x!1 are Skolem constants
  |-------
{1}   x!1 > x!2
[2]   re?(x!2)
```

- PVS represents current proof state as leaves of a tree of sequents (here there is just one leaf); each sequent has a collection of numbered formulas above and below the `|-----` turnstile line; interpretation is the conjunction of formulas above the line entail the disjunction of those below.
- Top level negations are eliminated by moving their formula to the other side of the turnstile, so equivalent. . .

Indirectly Begging the Question

- We start a PVS proof of God_re_alt, introduce premises ExUnd and Ex_re, expand definition of God?, perform routine steps of Skolemization, instantiation, and propositional simplification
- And we arrive at the following sequent

```
[-1]   re?(x!1)    % Terms such as x!1 are Skolem constants
[2]    NOT re?(x!2)
  |-------
{1}    x!1 > x!2
```

- PVS represents current proof state as leaves of a tree of sequents (here there is just one leaf); each sequent has a collection of numbered formulas above and below the |------ turnstile line; interpretation is the conjunction of formulas above the line entail the disjunction of those below.
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Indirectly Begging the Question (ctd.)

- If we ask PVS to generalize the Skolem constants, we get

```
FORALL (x_1, x_2:  beings):
  re?(x_2) IMPLIES x_2 > x_1 OR re?(x_1)
```

- Renaming the variables and rearranging, this is

```
FORALL (x, y:  beings):
  (re?(x) AND NOT re?(y)) IMPLIES x > y
```

- Which is identical to `Greater2`

- Thus, `Greater2` corresponds precisely to the formula required to discharge the final step of the proof

- Call that indirect begging the question

Why is This Begging the Question?

- By indirect begging, I mean
  - A premise that precisely discharges a key step of the proof
  - Not necessarily the final one
  - When previous steps are entirely routine
    - ⋆ i.e., no "entrapment"
- The sequent is a pretty good summary of our epistemic state after digesting the other premises
  - Provided no heavy-duty deduction
- So if the questionable premise fits it precisely, then it looks like reverse-engineering
- Content of the premise is entirely predictable
- So eliminates any hope of surprise or insight
- Hence I consider it question begging

Indirect Begging in More Complex Proofs

- E&R have a higher order version of the argument
- Beings have properties, and real existence is one of these
- One being is > than another if it has all its properties and more besides
- If the "something than which" does not really exist
- Then consider a being with same properties plus real existence
- Problem is, don't know there is such a being (in the domain of quantification/in the type/in the understanding)
- E&R provide premise `Realization` that says for any set of properties, there is a being with just those properties
- Eh? What if there are incompatible properties? . . . later

Indirect Begging in More Complex Proofs (ctd)

- I think `Realization` indirectly begs the question, but its use is lost in the larger proof
- Here's a PVS technique to expose it
- Also makes the strategy of the proof explicit
- "Consider a being with same properties plus real existence"
  `(name "X" "choose! z: FORALL F: F(z) = (F(x!1) OR F=re?)")`
- Here, `choose!` is a binder derived from choice function `choose`
- Whose argument must be nonempty, hence get this TCC
  `EXISTS (x: beings): (FORALL F: F(x) = (F(x!1) OR F = re?))`
- Cite `Realization` and instantiate its variable `FF` with the term
  `{ G: (P) | G(x!1) or G=re? }`
- And it provides exactly the expression above
- Hence it indirectly begs the question

Indirect Begging in More Complex Proofs (ctd 2)

- Campbell rejects `Realization`
  - Because (I think) of problem with incompatible properties
    - ⋆ E&R avoid inconsistencies by requiring positive properties
    - ⋆ But can still have incompatibilities:
      e.g., perfectly just vs. perfectly merciful

- He uses another construction from E&R

- And a premise that asserts we can always add real existence to a set of properties

- Can use the same technique to indict his premise of indirectly begging the question

Other Examples

- I have also examined modal versions of the argument
  - Due to Adams, E&R, Lewis, Rowe
- All indirectly beg the question
- Am documenting these in another paper

Observations on Indirect Begging

- Unless redundant or superfluous,
  all premises are essential to a proof
- So any premise might be considered indirectly question begging
- And proof might be manipulated to manifest this
- So indirect begging is not a smoking gun
- But used with judgement, it can suggest a crime scene

Begging Your Questions

- I propose three criteria for question begging in formal proofs
  - Strict, weak, and indirect
- What do you think of these three criteria?
- Detection of question begging requires exploration
- Mechanization reduces exploration to calculation
  - Fast and reliable
  - But do you think it is useful?
  - And feasible for non-CS people?
- Does demonstration of question begging reduce your confidence in the conclusion
- Or your interest in it and its proof?
- Any other questions or comments?
- Full paper at

  http://www.csl.sri.com/users/rushby/abstracts/ontargbegs17