Opportunities for Industrial Applications of Formal Methods

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Formal Methods

- These are ways for exploring properties of computational systems for all possible executions
- As opposed to testing or simulation
    - These just sample the space of executions
- Formal methods use symbolic methods of calculation, e.g.,
    - Abstract interpretation
    - Model checking
    - Theorem proving
- Cf. $x^2 - y^2 = (x - y)(x + y)$ vs. `5*5-3*3 = (5-3)*(5+3)`

Practical Formal Methods

- Symbolic calculations have high computational complexity
    - NP Hard or worse, often superexponential, sometimes undecidable
- So to make them practical we have to compromise
    - Accept some wrong answers
        - ⋆ Incompleteness (false alarms)
        - ⋆ Unsoundness (undetected bugs)
    - Consider only very simple properties (not full correctness)
    - Focus on models of the software, not the actual code
    - Use human guidance
- Let's look at some of these

Bug Finding by Static Analysis

- Many commercial tools are available for this
  - E.g., Coverity, KlocWork, CodeSonar, . . . FindBugs, . . . Lint
  - These work on C, C++, Java
- Most are tuned to reduce the number of false alarms
- Even at the cost of missing some real bugs (i.e., unsound)
- Because the main market is in bug finding

Example: Bug Finding by Static Analysis

```
unsigned int X, Y;
while (1) {
  /* ... */
  B = (X == 0);
  /* ... */
  if (B) {
    Y = 1 / X
  };
  /* ... */
};
```

```
int x, y, z;
y = 1;
while (1) {
  if (x > 0) {
    y = y+x
  } else {
    y = y-x
  }
  z = 1 / y
}
```

A simple static analyzer will find the bug on the left, but will probably give a false alarm for the correct program on the right

- Or else fail to find the bug when `y` is initialized to `0`

Verification by Static Analysis

- Some tools are tuned the other way
- Mostly for safety-critical applications
- Guarantee to find all bugs in a certain class (i.e., sound)
- Possibly at the cost of false alarms
- For example

  Spark Examiner: guarantee absence of runtime errors
  (e.g., divide by zero) in Ada

  Astrée guarantee no over/underflow or loss of precision in
  floating point calculations (in C generated from SCADE)

Example: Verification by Static Analysis

We abstract integers by their signs

```
int x, y, z;      x, y in {neg, zero, pos}
y = 1;            y is pos
while (1) {
  if (x > 0) {
    y = y+x       x is pos;  y ← pos ⊕ pos; i.e., pos
  } else {
    y = y-x       x ∈ {zero, neg};  y ← pos ⊖ {zero, neg},
  }                                             i.e., pos
  z = 1 / y       division is ok
}
```

This is an example of data abstraction; other methods include predicate abstraction, and abstract interpretation

Model Checking

- Most static analyzers consider only simple properties
    - Often the properties are built-in and fixed
    - E.g., range of values each variable may take
- Model checking is more versatile
- User can specify property
- There are model checkers for C and Java
- But most work on more abstract models of software (typically state machines)
- We'll do an example

Car Door Locking Example

- Highly simplified from an example by Philipps and Scholz
- Controller for door locks
  - To keep it simple, we'll have just one door
- The lock can be in one of four states:

  `locking, unlocking, locked, unlocked`

  Starts in the `unlocked` state
- At each time step it takes an input with one of three values

  `open, close, idle`

  And asserts a signal `ready` when it is `locked` or `unlocked`
- The controller receives the `ready` signal from the lock, a `crash` signal from the airbag, and a command from the user

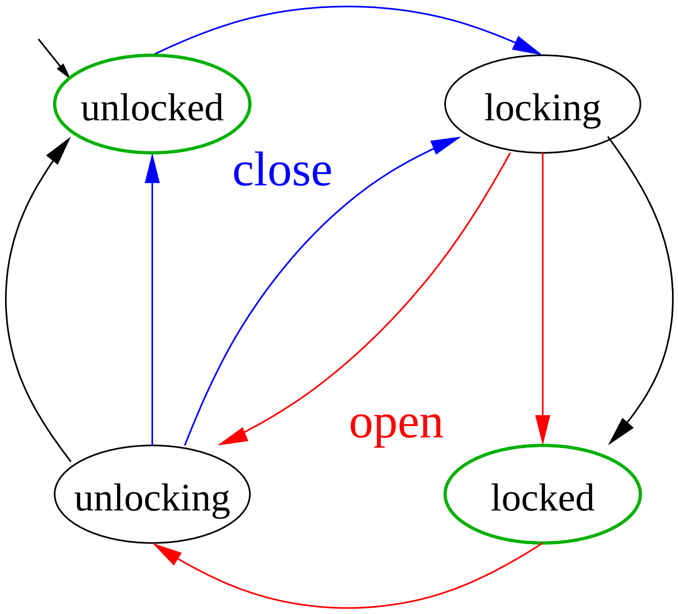
  `open, close, idle`
- Safety requirement:
  - Door is unlocked following open command, or crash

Car Door Locking Example (ctd)

- The lock is given, it behaves as follows
  - When it receives a `close` input:
    - ⋆ Does nothing if already `locked`
    - ⋆ If it is `unlocked`, goes to the intermediate `locking` state
    - ⋆ If it is `locking`, goes to `locked`
    - ⋆ If it is `unlocking`, nondeterministically continues to `unlocked`, or reverses to `locking`
  - Mutatis mutandis for `open` input
  - See state machine on next page
- Our task is to design the controller
  - Lock may still be performing a previous action
  - Only visibility into the lock's state is the `ready` signal
  - Which it sees with one cycle delay

Lock and Controller

Lock (given)



Output `ready` in green states

Controller (designed)

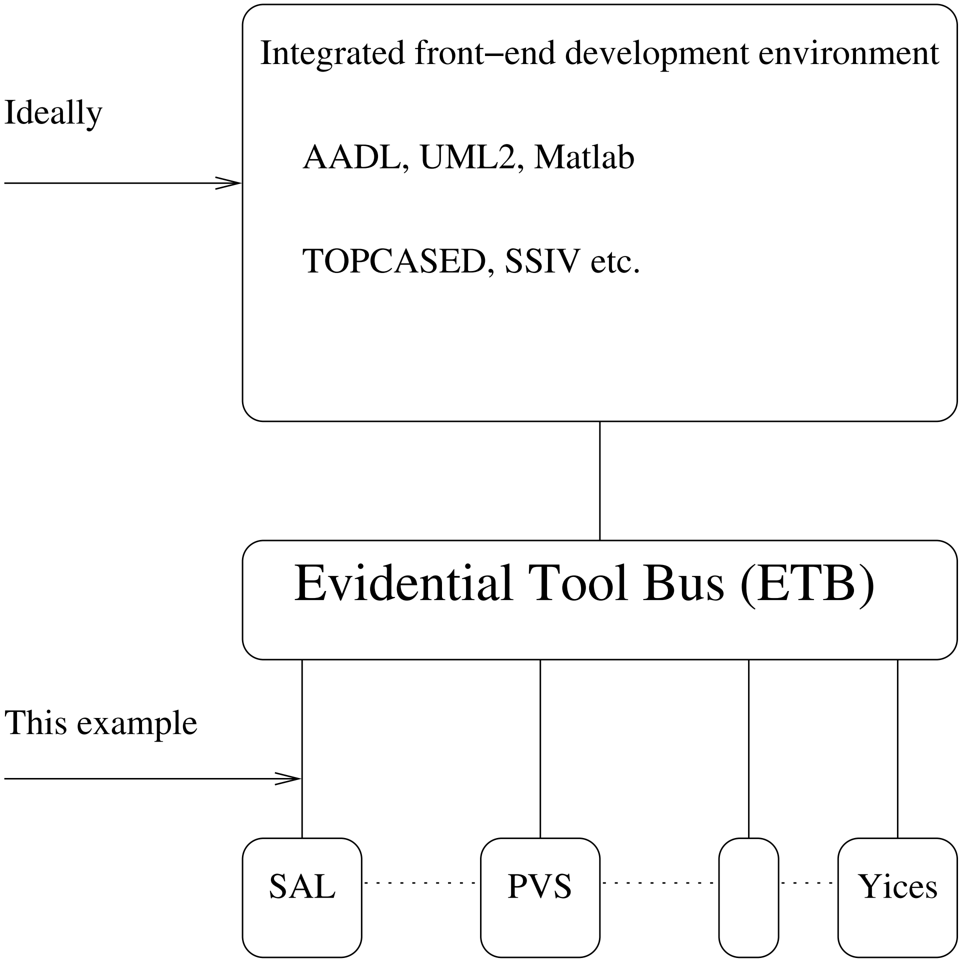
Inputs	output
crash	open
open	open
close	close
idle & ready	idle
else	repeat last

Model Checking the Car Door Locking Example

- Typically, we would specify this in a statecharts-like graphical formalism (e.g., StateFlow)
- But I will use the textual input to the SAL model checkers so we can see more of what is going on
- It's fairly easy to build translators and GUIs from engineering notations to the raw notation of a model checker

The Car Door Locking Example: Model Checker Input

Ideally, use an integrated front end; here we look at raw model-checker input



Integrated front–end development environment

Ideally

AADL, UML2, Matlab

TOPCASED, SSIV etc.

Evidential Tool Bus (ETB)

This example

SAL

PVS

Yices

Beginning of the `lock` Module in SAL

```
lock: MODULE =
BEGIN
INPUT
 action: lockaction
OUTPUT
 ready: BOOLEAN
LOCAL
 state: lockstate
INITIALIZATION
 state = unlocked
DEFINITION
 ready = (state = locked OR state = unlocked);
TRANSITION
[
locking:
 action = close AND state = unlocked --> state' = locking;
[]
reverse_unlocking:
 action = close AND state = unlocking -->
   state' IN {s: lockstate | s = locking OR s = unlocked}
```

Rest of the `lock` Module in SAL

```
[]
lock:
 state = locking --> state’ = locked;
[]
unlocking:
 action = open AND state = locked --> state’ = unlocking;
[]
reverse_locking:
 action = open AND state = locking -->
    state’ IN {s: lockstate | s = unlocking OR s = locked}
[]
unlock:
 state = unlocking --> state’ = unlocked;
[]
 ELSE -->
]
END;
```

Beginning of the `controller` Module in SAL

```
controller: MODULE =
BEGIN
INPUT
  user: lockaction,
  ready:  BOOLEAN,
  crash: BOOLEAN
OUTPUT
  action: lockaction
INITIALIZATION
  action = idle;
```

Rest of the `controller` Module in SAL

```
TRANSITION
[
crash:
 crash --> action' = open;
[]
open:
 user = open --> action' = open;
[]
close:
 user = close --> action' = close;
[]
return_to_idle:
 user = idle AND ready --> action' = idle;
[]
 ELSE -->
]
END;
```

Specifying The System and a Property

- The `system` is the synchronous composition of the two modules

  ```
  system: MODULE = lock || controller;
  ```

- Inputs and outputs with matching names (i.e., `lockaction` and `ready`) are automatically “wired up”
- Now we’ll check a property: whenever the `user` gives an `open` input, then the state will eventually be `unlocked`
  - We need to be careful that the user doesn’t immediately cancel the `open` with a `close`
  - So we’ll require that there are no `close` inputs following the `open`
- We could have GUI for specifying properties, but here we’ll use Linear Temporal Logic (LTL) which is the raw input to a model checker

A Formal Analysis

- We specify the property in LTL as follows:

```
prop1: LEMMA system |-
 G(user=open AND X(G(user /= close)) => F(state=unlocked));
```

- In LTL, `G` means always, `F` means eventually, and `X` means next state
    - These are sometimes written □, ◇, and ∘, respectively
- We put all the SAL text into a file `door.sal`
- Then we can ask the SAL **symbolic** model checker to check the property `prop1`:

```
  sal-smc -v 3 door prop1
```

- In a fraction of a second it says: proved
- Unlike a simulation, this has considered all possible scenarios satisfying the hypothesis (e.g., whether lock is `ready` or not).

More Analyses

- We can check that the door eventually always stays unlocked

```
prop1a: LEMMA system |-
 G(user = open AND X(G(user /= close))
     => F(G(state = unlocked)));
```

- And we can sharpen eventually to four steps

```
prop1b: LEMMA system |-
 G(user=open AND X(G(user/=close))
     => XXXX(G(state = unlocked)));
```

(`XXXX` is a macro for four applications of `X`)

- We can check that four is the minimum by trying three

```
prop1c: LEMMA system |-
 G(user=open AND X(G(user/=close))
     => XXX(state = unlocked));
```

- Sure enough, SAL says invalid

Counterexamples

- But it also gives us a counterexample

```
user  :  close    open     idle    idle
action:           close    open    idle   idle
state :  unlocked unlocked locking locked unlocking unlocked
```

- Push-button proof is nice, but counterexamples are a major additional benefit of model checking: when a property is invalid, we get a trace that manifests its invalidity
- For example, let's check that the `crash` input always results in the door becoming `unlocked`
- We'll start by assuming the user does no `close` inputs when the `crash` occurs
-
```
prop2: LEMMA system |-
 G(crash AND G(user /= close) => F(state = unlocked));
```

Another Counterexample

- SAL says invalid and the counterexample shows that the `crash` input occurs when the door is `locked` and the guard on the `return_to_idle` transition is enabled. . . and the system chooses to take the latter transition

- We need to add `NOT crash` to the guard for the `return_to_idle` transition to ensure it cannot occur when `crash` is enabled

- Now `prop2` is proved

Yet Another Counterexample

- Next, let's check whether we can allow a `close` input when the crash occurs
- `prop3: LEMMA system |-`
  ```
   G(crash AND X(G(user /= close)) => F(state = unlocked));
  ```
- We get another counterexample!
- A fix is to add `NOT crash` to the guard for the `close` transition, too

Counterexamples And Test Case Generation

- We can generate test cases by providing deliberately false assertions
- The counterexample is a test case
- To get a test case that drives the system to a state where property P is true, use the property `G(not P)`
- Example: test case to get the system into the unlocking state

  ```
  test1: LEMMA system |- G(state /= unlocking);
  ```
- The test case is the is the input sequence `close`, `open`

Model Checking Technology

- Technically, a model checker tests whether a system specification is a Kripke model of a property expressed as temporal logic formula
- The simplest kind of property is an invariant (`G(p)` in LTL)
  - i.e., one that is true in every reachable state
- So the simplest kind of model checking is reachability analysis
- Construct every reachable state of the system and check that desired properties (invariants) hold
  - Feasible if all state variables are finite
  - May require abstraction to achieve this
- Simplest method: explicit state reachability analysis
  - E.g., SPIN

Explicit State Reachability Analysis and Model Checking

- Imagine a simulator for some system/environment model
- Keep a set of all states visited so far, and a list of all states whose successors have not yet been calculated
  - Initialize both with the initial states
- Pick a state off the list and calculate all its successors
  - i.e., run all possible one-step simulations from that state

  Throw away those seen before
- Add new ones to the set and the list
- Check each new state for the desired properties
- Iterate to termination, or some state fails the property
  - Or run out of memory, time, patience
- On failure, counterexample (backtrace) manifests problem
- Extend to model checking of general LTL properties using Büchi automata

Symbolic Model Checking

- Explicit state model checkers run out of power around 10-100 million reachable states
- But that's only around 25 state bits
- Can often represent states more compactly using symbolic representation
- E.g., the infinite set of states $\{(0,1),(0,2),(0,3),\ldots(1,2),(1,3),\ldots(2,3),\ldots\}$ can be symbolically represented as the finite expression $\{(x,y)\,|\,x<y\}$
- Symbolic model checkers use such symbolic representations
-
    - E.g. NuSMV, sal-smc

Symbolic Model Checking (ctd)

- Compile the model to a Boolean transition relation T
  - i.e., a circuit
- Initialize the Boolean representation of the stateset S to the initial states I
- Repeatedly apply T to S until a fixpoint
  - $S' = S \cup \{t \,|\, \exists s \in S : T(s, t)\}$
  - Final S is a formula representing all the reachable states
- Check the property against final S
- Mechanized efficiently using BDDs
  - Reduced ordered Binary Decision Diagrams

Commodity software, honed by competition (CUDD)

Bounded Model Checking

- Modern symbolic model checkers can handle 600 state bits before special tricks are needed
- seldom get beyond 1,000 state bits
- Bounded model checkers are specialized to finding counterexamples
- Sometimes can handle bigger problems than SMC
    - E.g, NuSMV, sal-bmc

Bounded Model Checking

- Is there a counterexample to P in k steps or less?
- Find assignments to states $s_0, \ldots, s_k$ such that

$$I(s_0) \wedge T(s_0, s_1) \wedge T(s_1, s_2) \wedge \cdots \wedge T(s_{k-1}, s_k) \wedge \neg(P(s_1) \wedge \cdots \wedge P(s_k))$$

- Given a Boolean encoding of I, T, and P (i.e., circuit), this is a propositional satisfiability (SAT) problem
- SAT is the quintessential NP-Complete problem
- But current SAT solvers are amazingly fast
- Commodity software, honed by competition (MiniSAT, Siege, zChaff, Berkmin)
- BMC uses same representation as SMC, different backend

Verification with BMC

- BMC was originally developed for refutation (bug finding)
- But can be used for verification of invariants via k-induction
- 1-induction; ordinary inductive invariance (for P):

  Basis: $I(s_0) \supset P(s_0)$

  Step: $P(r_0) \wedge T(r_0, r_1) \supset P(r_1)$
- Extend to induction of depth k (cf. strong induction):

  Basis: No counterexample of length k or less

  Step: $P(r_0) \wedge T(r_0, r_1) \wedge P(r_1) \wedge \cdots \wedge P(r_{k-1}) \wedge T(r_{k-1}, r_k) \supset P(r_k)$

  These are close relatives of the BMC formulas
- Induction for $k = 2, 3, 4 \ldots$ may succeed where $k = 1$ does not
  - Can also use lemmas
- Note that counterexamples help debug invariant

SAT Solving

- Find satisfying assignment to a propositional logic formula
- Formula can be represented as a set of clauses
  - In CNF: conjunction of disjunctions
  - Find an assignment of truth values to variable that makes at least one literal in each clause TRUE
  - Literal: an atomic proposition A or its negation $\bar{A}$
- Example: given following 4 clauses
  - A, B
  - C, D
  - E
  - $\bar{A}, \bar{D}, \bar{E}$

  One solution is $A, C, E, \bar{D}$

  (A, D, E is not and cannot be extended to be one)
- Do this when there are 1,000,000s of variables and clauses

SAT Solvers

- SAT solving is the quintessential NP-complete problem
- But now amazingly fast in practice (most of the time)
    - Breakthroughs (starting with Chaff) since 2001
        - Building on earlier innovations in SATO, GRASP
    - Sustained improvements, honed by competition
- Has become a commodity technology
    - MiniSAT is 700 SLOC
- Can think of it as massively effective search
    - So use it when your problem can be formulated as SAT
- Used in bounded model checking and in AI planning
    - Routine to handle 10^{300} states

SAT Plus Theories

- SAT can encode operations on bounded integers
  - Using bitvector representation
  - With adders etc. represented as Boolean circuits

  And other finite data types and structures
- But cannot do not unbounded types (e.g., reals), or infinite structures (e.g., queues, lists)
- And even bounded arithmetic can be slow when large
- There are fast decision procedures for these theories
- But their basic form works only on conjunctions
- General propositional structure requires case analysis
  - Should use efficient search strategies of SAT solvers

  That's what a solver for Satisfiability Modulo Theories does
  - SMT solvers: e.g., Barcelogic, CVC, MathSAT, Yices
  - Sustained improvements, honed by competition

Decidable Theories

- Many useful theories are decidable
  (at least in their unquantified forms)
  - Equality with uninterpreted function symbols
    $x = y \wedge f(f(f(x))) = f(x) \supset f(f(f(f(f(y))))) = f(x)$
  - Function, record, and tuple updates
    $f \textbf{ with } [(x) := y](z) \stackrel{\text{def}}{=} \textbf{if } z = x \textbf{ then } y \textbf{ else } f(z)$
  - Linear arithmetic (over integers and rationals)
    $x \le y \wedge x \le 1 - y \wedge 2 \times x \ge 1 \supset 4 \times x = 2$
  - Special (fast) case: difference logic
    $x - y < c$
- Combinations of decidable theories are (usually) decidable

$e.g.,$ $2 \times car(x) - 3 \times cdr(x) = f(cdr(x)) \supset$

$$f(cons(4 \times car(x) - 2 \times f(cdr(x)), y)) = f(cons(6 \times cdr(x), y))$$

Uses equality, uninterpreted functions, linear arithmetic, lists

SMT Solving

- Individual and combined decision procedures decide conjunctions of formulas in their decided theories
- SMT allows general propositional structure
  - e.g., $(x \leq y \vee y = 5) \wedge (x < 0 \vee y \leq x) \wedge x \neq y$

    . . . possibly continued for 1000s of terms
- Should exploit search strategies of modern SAT solvers
- So replace the terms by propositional variables
  - i.e., $(A \vee B) \wedge (C \vee D) \wedge E$
- Get a solution from a SAT solver (if none, we are done)
  - e.g., A, D, E
- Restore the interpretation of variables and send the conjunction to the core decision procedure
  - i.e., $x \leq y \wedge y \leq x \wedge x \neq y$

SMT Solving by "Lemmas On Demand"

- If satisfiable, we are done
- If not, ask SAT solver for a new assignment
- But isn't it expensive to keep doing this?
- Yes, so first, do a little bit of work to find fragments that explain the unsatisfiability, and send these back to the SAT solver as additional constraints (i.e., lemmas)
  - $A \wedge D \supset \bar{E}$ (equivalently, $\bar{A} \vee \bar{D} \vee \bar{E}$)
- Iterate to termination
  - e.g., $A, C, E, \bar{D}$
  - i.e., $x \leq y, x < 0, x \neq y, y \not\leq x$ (simplifies to $x < y, x < 0$)
  - A satisfying assignment is $x = -3, y = 1$
- This is called "lemmas on demand" (de Moura, Ruess, Sorea) or "DPLL(T)"; it yields effective SMT solvers

Infinite Bounded Model Checking

- These are bounded model checkers that use SMT solvers
    - E.g., sal-inf-bmc
- Allow analysis of models with infinite state spaces
    - E.g., real-time, other continuous variables

Model Checking for Hybrid Systems

- Often need plant models with continuous dynamics
  - i.e., differential equations
- Hybrid systems mix discrete and continuous behavior
  - As in Simulink/StateFlow
  - Timed systems are a special case
- There are specialized model checkers for hybrid systems
  - E.g., Checkmate

  Seldom get beyond 5 or 6 continuous variables
- Another approach uses automated theorem proving to abstract hybrid systems to conservative discrete approximations
  - E.g., hybrid-sal

  Can sometimes handle 25 continuous variables

The Ecosystem of Formal Methods Tools

- Underlying technology is highly competitive, specialized
  - Abstract interpreters, BDDs, SAT, SMT solvers, general theorem proving
- Next level is well-understood, established incumbents
  - Static analyzers, model checkers, full theorem provers
- The action is in automation of the outer loop
  - Counterexample-guided abstraction refinement, interpolants

  And specialized combinations
  - Mixed concrete and symbolic (concolic) execution
  - Combinations of methods
    - ⋆ Static analysis generates lemmas for model checker
- The opportunities are in enabling these combinations
  - Tool buses: open up the tools, make them scriptable

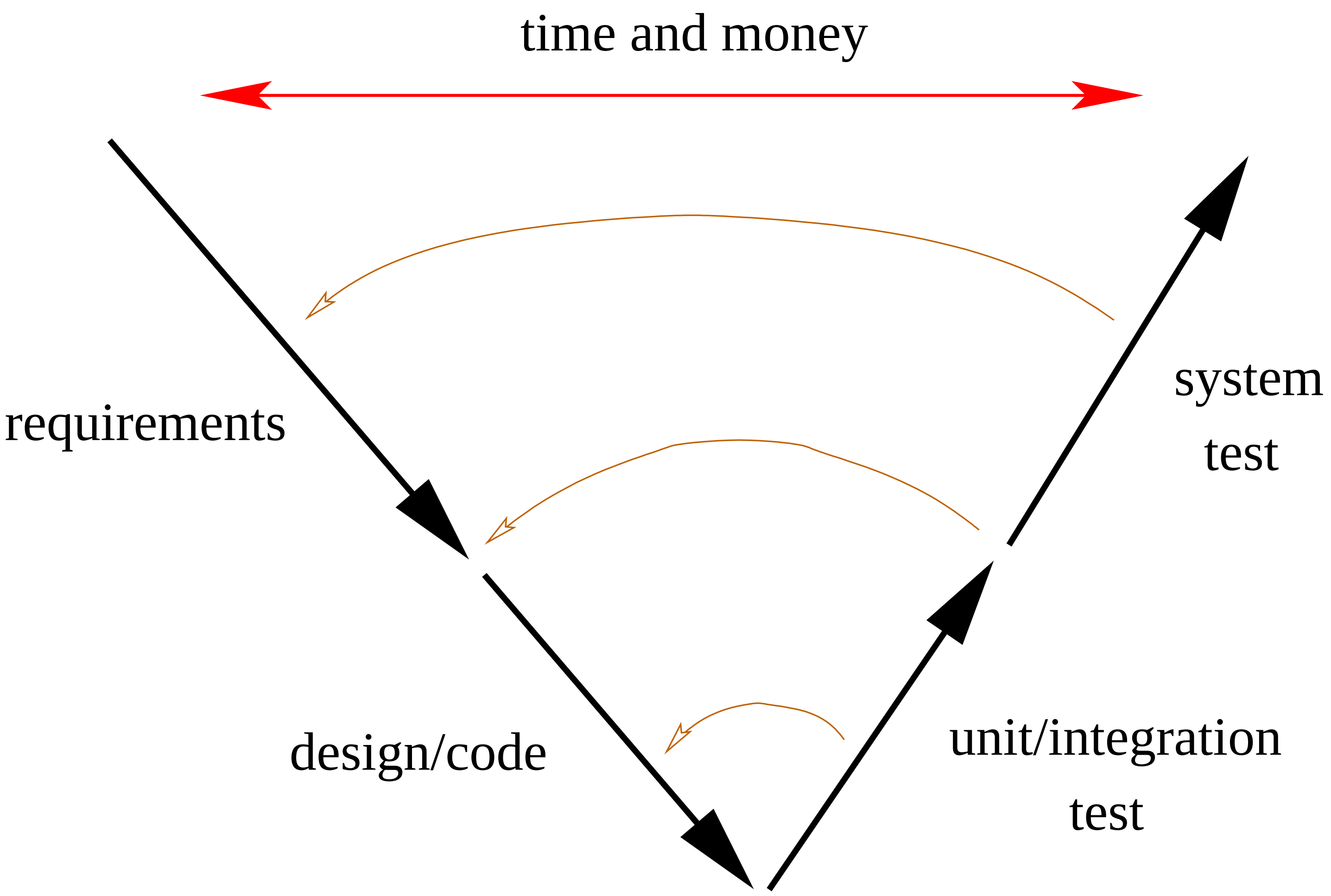
Integration Example: LAST

- LAST (Xia, DiVito, Muñoz) generates MC/DC tests for avionics code involving nonlinear arithmetic (with floating point numbers, trigonometric functions etc.)
- Applied it to Boeing autopilot simulator
- Generated tests to (almost) full MC/DC coverage in minutes
- It's built on Blast (Henzinger et al)
    - ○ A software model checker, itself built of components
    - ○ Including CIL and CVC-Lite
- But extends it to handle nonlinear arithmetic using RealPaver (a numerical nonlinear constraint unsatisfiability checker)
    - ○ Added 1,000 lines to CIL front end for MC/DC
    - ○ Added 2,000 lines to integrate RealPaver with CVC-Lite
    - ○ Changed 2,000 lines in Blast to tie it all together
- Toolbus goal is to simplify this kind of construction

Opportunities for Applications of Formal Methods

- The ability of formal methods to consider all possible executions creates powerful opportunities
- Exploration of properties in early-lifecycle models
- Thorough analysis of detailed design models
- Guaranteed detection of certain classes of errors in implementations
- Automated generation of test cases

Traditional Vee Diagram (Much Simplified)



time and money

requirements

system

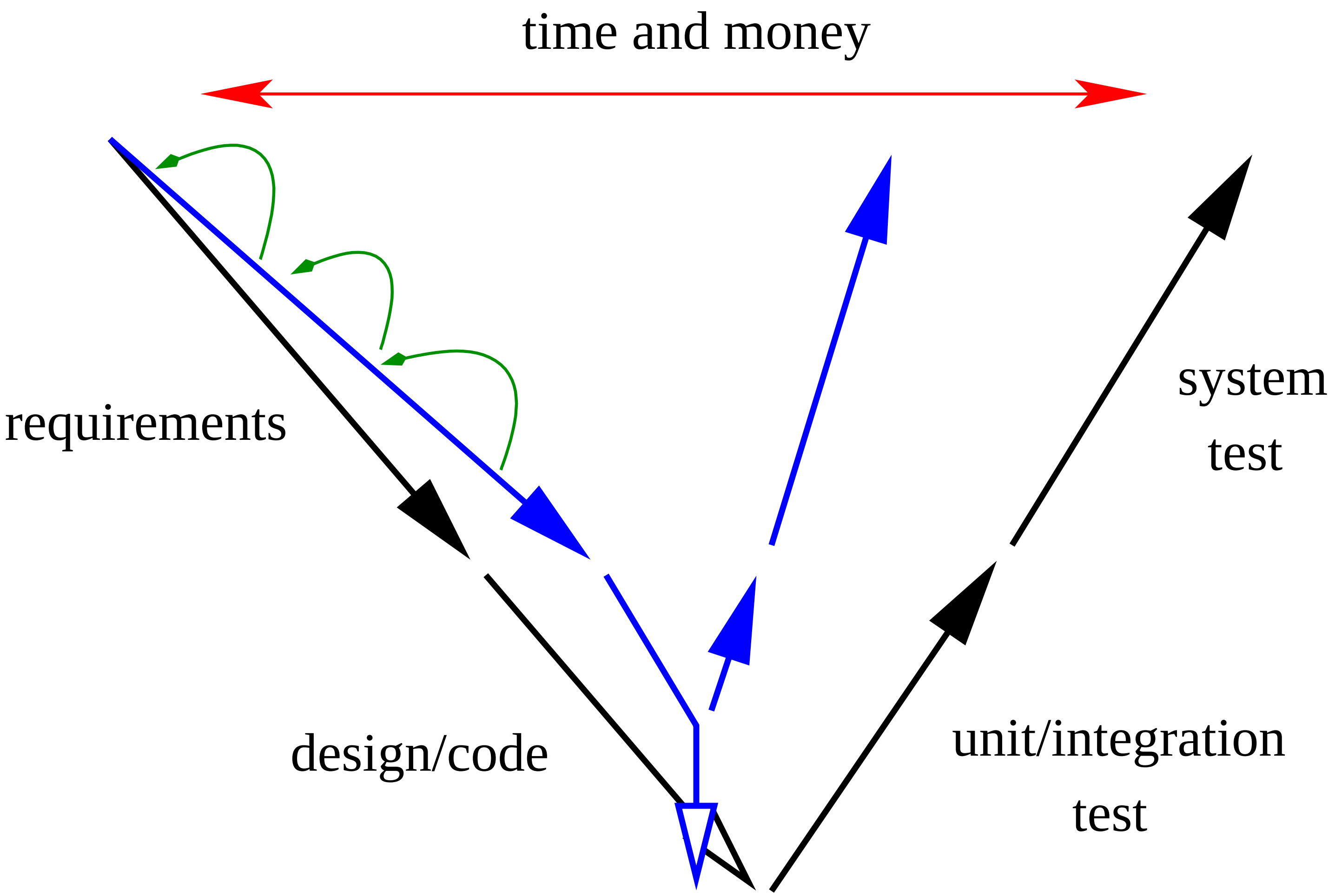
test

design/code

unit/integration

test

Vee Diagram Tightened with Formal Methods



Example: Rockwell-Collins

Industrial Applications of Formal Methods

- Need to integrate formal methods in the development tool-chain
    - Interfacing different notations
    - Automating/assisting abstraction and lemma generation
- Do so in an open-ended way that allows new tools
- And combinations of tools
- Get in early
    - Pick the low-hanging fruit

  Ride the wave of increasing power as the technology matures
- Good luck!