Architecture, Arguments, and Confidence

(Joint work with Bev Littlewood, City University, London UK)

John Rushby

Computer Science Laboratory
SRI International
Menlo Park CA USA
Overview

• Many assurance cases involve quantification of risk
• Which in turn requires quantifying failure rates of software
• Notoriously hard to do, beyond about $10^{-3}$
  ○ Which you can test for
• So to provide assessments for higher reliabilities, either need very strong analysis
  ○ Viewed skeptically by some: e.g., CAST 24
• Or software redundancy
• And that requires choices about the software architecture, the kinds of claims, and the types of argument that can support an assurance case that involves software redundancy

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Overview (ctd.)

- I’ll outline an approach that combines consideration of architecture, claims about formal verification, and novel probabilistic reasoning
- Will apply it first to one-out-of-two architectures of the kind used for nuclear shutdown
- Then to monitored architectures of a kind proposed for aircraft (software IVHM)
Reliability of Redundant and Monitored Systems

- It is well-known that the reliability of systems with redundant software channels cannot be estimated simply by multiplying the reliabilities of their constituent channels.

- Empirical and theoretical studies confirm that failures may not be independent.
  - Even when channels are deliberately diverse.
  - Some situations are intrinsically more difficult.

- Littlewood and Miller model gives probability of system failure as $pfd_A \times pfd_B + Cov(\theta_A, \theta_B)$ where $\theta_A, \theta_B$ are the difficulty function random variables for the two channels.

- Hard to estimate these, and their covariance.

- Same considerations apply when we have an operational (sub)system and a monitor.
Reliability of Systems With a Possibly-Perfect Monitor

• But suppose the claim we make for the monitor is not that it achieves some particular reliability
  ○ i.e., has some probability of failure on demand

• But that it is possibly perfect
  ○ Will need to be simple, and have very strong assurance

• Perfect means that it will never experience a failure

• Possibly perfect means there is some uncertainty about its perfection
  ○ In particular, it has a probability of imperfection

• We need to be careful about the uncertainties and probabilities here
Aleatory and Epistemic Uncertainty

- **Aleatory** or **irreducible** uncertainty
  
  - is “uncertainty in the world”
  
  - e.g., if I have a biased coin with $P(\text{heads}) = p_h$, I cannot predict exactly how many heads will occur in 100 trials because of randomness in the world

  *Frequentist* interpretation of probability needed here

- **Epistemic** or **reducible** uncertainty
  
  - is “uncertainty about the world”
  
  - e.g., if I give you the biased coin, you will not know $p_h$; you can estimate it, and can try to improve your estimate by doing experiments, learning something about its manufacture, the historical record of similar coins etc.

  *Frequentist* and *subjective* interpretations OK here
Aleatory and Epistemic Uncertainty in Models

- In much scientific modeling, the aleatory uncertainty is captured conditionally in a model with parameters.
- And the epistemic uncertainty centers upon the values of these parameters.
- As in the coin tossing example.
One Out Of Two (1oo2) Architectures

- These are systems, like those used for nuclear shutdown, that have two dissimilar channels in parallel
- Either can shut the system down (no voting)
- So system failure requires both channels to fail
- Suppose one is a complex, but highly reliable system $A$, with aleatory probability of failure on demand ($p_{fd}$) $p_A$
- And suppose the other is a simple system $B$ that is possibly perfect with aleatory probability of imperfection ($p_{np}$) $p_B$
  - One way to give this a frequentist interpretation is to consider all the channels that might have been developed by the same process, and then consider the proportion of those that are imperfect
- Note that we are assuming $p_A$ and $p_B$ are known
- What is the probability of system failure?
Aleatory Uncertainty for 1oo2 Architectures

\[
P(\text{system fails} \mid \text{on randomly selected demand}) = \frac{p_{fd A} p_{np B}}{p_{np B}}
\]

Assume, conservatively, that if \( A \) fails and \( B \) is imperfect, then \( B \) will fail on the same demand.

\[
\leq 1 \times P(A \text{ fails, } B \text{ imperfect} \mid p_{fd A} = p_A, p_{np B} = p_B) + 0 + 0 + 0
\]
Aleatory Uncertainty for 1oo2 Architectures (ctd.)

\[
P(A \text{ fails, } B \text{ imperfect} \mid pfd_A = p_A, pnp_B = p_B) \]

\[
= P(A \text{ fails} \mid B \text{ imperfect, } pfd_A = p_A, pnp_B = p_B) \times P(B \text{ imperfect} \mid pfd_A = p_A, pnp_B = p_B) 
\]

(Im)perfection of \( B \) tells us nothing about the failure of \( A \) on this demand; hence,

\[
= P(A \text{ fails} \mid pfd_A = p_A, pnp_B = p_B) \times P(B \text{ imperfect} \mid pfd_A = p_A, pnp_B = p_B) 
\]

\[
= p_A \times p_B 
\]

Compare with two (un)reliable channels, where failure of \( B \) on this demand does increase likelihood \( A \) will fail on same demand

\[
P(A \text{ fails} \mid B \text{ fails}, pfd_A = p_A, pfd_B = p_B) \]

\[
\geq P(A \text{ fails} \mid pfd_A = p_A, pfd_B = p_B) 
\]
Aleatory Uncertainty for 1oo2 Architectures (ctd. 2)

I could have factored the conditional probability involving the perfect channel the other way around:

\[
P(A \text{ fails}, B \text{ imperfect} \mid pfd_A = p_A, pnp_B = p_B) \\
= P(B \text{ imperfect} \mid A \text{ fails}, pfd_A = p_A, pnp_B = p_B) \\
\times P(A \text{ fails} \mid pfd_A = p_A, pnp_B = p_B)
\]

You might say knowledge that \(A\) has failed should affect my estimate of \(B\)'s imperfection, but we are dealing with aleatory uncertainty where these probabilities are known; hence

\[
= P(B \text{ imperfect} \mid pfd_A = p_A, pnp_B = p_B) \\
\times P(A \text{ fails} \mid pfd_A = p_A, pnp_B = p_B) \\
= p_B \times p_A \text{ as before}
\]

Note: the claim must be perfection, other global properties (e.g., proven correct) are not aleatory (they are reducible)
• We have shown that the events "A fails" "B is imperfect" are conditionally independent at the aleatory level

• Knowing aleatory probabilities of these allows probability of system failure to be conservatively bounded by $p_A \times p_B$

• But we do not know $p_A$ and $p_B$ with certainty: assessor formulates beliefs about these as subjective probabilities

• The beliefs may not be independent, so they will be represented by a joint probability density function

$$dF(p_A, p_B) = P(pfd_A < p_A, pnp_B < p_B)$$

• The unconditional probability of system failure is then

$$P(\text{system fails on randomly selected demand}) = \int_{0 \leq p_A \leq 1} \int_{0 \leq p_B \leq 1} p_A \times p_B \, dF(p_A, p_B)$$

(That’s a Riemann-Stieltjes integral)
Reliability Estimate for 1oo2 Architectures

- The only source of dependence is in the assessor’s bivariate density function \( dF(p_A, p_B) \)
- But it is really hard to elicit such bivariate beliefs
- What stops beliefs about the two parameters being independent?
- It’s not difficulty variation over the demand space
  - Formal verification is uniformly credible
- Surely, it’s concern about common-cause errors such as misunderstood requirements, common mechanisms, etc.
- So combine all beliefs about common-cause faults in a third parameter \( C \)
  - Place probability mass \( C \) at point \((1, 1)\) in \((p_A, p_B)\)-plane as subjective probability for such common faults
Reliability Estimate for 1oo2 Architectures (ctd.)

- With probability $C$, $A$ will fail with certainty, and $B$ will be imperfect with certainty (and conservatively assumed to fail).

- If assessor believes all dependence between his beliefs about the model parameters has been captured conservatively in $C$, the conditional distribution factorizes, so

\[
P(\text{system fails on randomly selected demand}) = C + (1 - C) \times \int_{0 \leq p_A < 1} p_A \, dF(p_A) \times \int_{0 \leq p_B < 1} p_B \, dF(p_B)
\]

\[= C + (1 - C) \times P_A^* \times P_B^*
\]

where $P_A^*$ and $P_B^*$ are the means of the marginal distributions excluding $(1, 1)$.
Reliability Estimate for 1oo2 Architectures (ctd. 2)

- If $C$ is small (as will be likely), can approximate as

$$C + P_A \times P_B$$

where $P_A$ and $P_B$ are the means of the marginal distributions.

- Construct probability $C$ by considering top-level development
  - Or by claim limits ($10^{-5}$)

- Construct probability $P_A$ by statistically valid random testing ($10^{-3}$)

- Construct probability $P_B$ by considering mechanically checked formal verification (see later) ($10^{-3}$)

- Hence overall system $pfd$ is about $1.1 \times 10^{-5}$
Failures of Commission

- Focus so far is failure of omission
  - e.g., not shutting down reactor when you should

- Also need to consider failures of commission
  - i.e., shutting down reactor when you should not
  - Failure of either channel can do this

- Failures of commission can be mere nuisances, have economic cost, or be safety-critical

- Have to be careful about demands (points in time) vs. nondemands (absence of demands over intervals of time)

- Discretize time: e.g., single flight of an aircraft

- Can then use pfd$s for both demands and nondemands
Failures of Commission

- By similar arguments as before, get

\[
P(\text{system fails on randomly selected non-demand} \mid \text{pf}d_A = p_{A2}, \text{pp}n p_B = p_{B2})
= p_{A2} + p_{B2} - p_{A2} \times p_{B2}
\]

- where \( p_{A2} \) and \( p_{B2} \) are aleatory probabilities of failure and imperfection, respectively, for \( A \) and \( B \) wrt. failures of commission

- This result shows us that the diversity in a 1oo2 architectures provides no benefit with respect to these failures

- For epistemic assessment, conservative to ignore final term, do not then need a factoring argument for epistemic values

- So system \( \text{pf}d \) wrt. failures of commission is \( P_{A2} + P_{B2} \) where \( P_{A2} \) and \( P_{B2} \) are means of the marginal distributions
Risk of Failures

- Denote the consequence (cost) of a failure of omission by $c_1$, and the consequences of failures of commission by the $A$ and $B$ channels by $c_{A2}$ and $c_{B2}$, respectively.
  - The costs are different because the two channels may operate in different ways.

- Denote the probability that a randomly selected interval triggers a demand by $f$.

- Then epistemic risk is bounded by

$$f \times c_1 \times (C + P_{A1} \times P_{B1}) + (1 - f) \times c_{A2} \times P_{A2} + (1 - f) \times c_{B2} \times P_{B2}$$

  omission + commission
Assurance Case for Formal Verification

• How might we construct probabilities $P_{B1}, P_{B2} \leq 10^{-3}$?

• i.e., less than 1 in 1,000 chance that the monitor is imperfect

• We will formally verify or formally synthesize the monitor
  ◦ i.e., prove it correct using automated tools

• What are the dominant hazards to this process?
  ◦ Topics outside formal analysis (e.g., compiler bugs)—those have to be included in $C$
    ★ Can be verified by testing (autogenerated from specs)
  ◦ Incorrect claims—that’s dealt with in $C$, too
  ◦ Incorrect formalization of claims and supporting theories
  ◦ Unsound formalization of these (e.g., flawed axioms)
  ◦ Unsound theorem prover or monitor synthesis
Soundness Guarantees for Formal Verification

- Unsound axiomatizations can be eliminated by constructive methods, or by exhibiting a constructive model.

- Of the remaining hazards, incorrect formalization of the claims and theories are surely dominant.
  - Allocate most of our $10^{-3}$ “budget” here.

- Then, an adequate soundness guarantee for our theorem prover or formal synthesis procedure will be about $10^{-4}$.

- This is not a very demanding requirement.
Soundness Guarantees for Formal Verification Tools

- A verification will certainly fail if your tools and deductive components lack the power to complete it.
- We need ways to guarantee soundness that do not compromise deductive power.
- Many options: computational reflection, diverse verifiers, trusted core, proof generation and verified checker.
- Computational reflection is fine, but has to build on something more basic.
- Diversity has well-known weaknesses.
- Trusted core is slow, and a weak guarantee.
  - Even the relatively solid and small (∼400 lines of OCaml) HOL Light core was found to have two soundness bugs.
  - Has since been (self) verified.

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Proof Generation and Verified Checkers

- Traditional approach is to generate primitive proof objects that can be independently checked by a verified proof kernel
  - An instance of an operational system with a monitor!

- Problem is the primitive proof objects from powerful provers (e.g., SMT solvers) are vast (gigabytes)

- We favor more powerful checkers and offline verifiers that can be driven by more succinct certificates and hints, respectively
  - Developing and formally verifying useful checkers and offline verifiers is a major research challenge
  - A high-performance SAT solver is a good start: checking of many verifiers can be reduced to SAT plus something
  - Shankar and Marc Vaucher have verified a modern SAT solver in PVS; the formal specification is efficiently executable (modulo lacunae in the PVS evaluator)
Verified Reference Kernels

Untrusted Frontline Verifier

Hints

Verified Offline Verifier

Certificates

Verified Checker

Proofs

Trusted Proof Kernel

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Software IVHM for Aircraft

- Requirements for safety critical software in aircraft are extreme (e.g., probability of failure $10^{-9}$/hour)

- Retrospective evidence it was achieved
  - At least, until recent accidents and incidents
  - A330 accident near Perth, 777 incident near Perth, A340 incident near Schiphol, 737 crash at Schiphol

- But how to assess it prospectively, in certification?

- Skepticism it can be achieved by analysis alone
  - e.g., CAST 24 report: suggests diversity

- IVHM is Integrated Vehicle Health Maintenance
  - Monitoring, prognosis, mitigation etc.

- Software IVHM applies this to software
A Recent Incident Due to Software

- An Airbus A340 en-route from Hong Kong to London on 8 February 2005

- Toward the end of the flight, two engines flamed out, crew found certain tanks were critically low on fuel, declared an emergency, landed at Amsterdam

- Two Fuel Control Monitoring Computers (FCMCs) on this type of airplane; they cross-compare and the “healthiest” one drives the outputs to the data bus

- Both FCMCs had fault indications, and one of them was unable to drive the data bus

- Unfortunately, this one was judged the healthiest and was given control of the bus even though it could not exercise it

- Further backup systems were not invoked because the FCMCs indicated they were not both failed
Software Health Management and Monitoring

• System hazards due to software faults are a topic of concern in aviation safety: one accident, and several serious incidents

• Traditional approach is fault avoidance
  ◦ Strive to eliminate software faults
  ◦ The intent of DO-178B, DO-297, etc.
  May be reaching the limits of effectiveness

• So consider buttressing it by software health management
  ◦ Techniques for monitoring, diagnosing, prognosing, and mitigating the manifestations of residual faults.

• But what specifications do we monitor against?
  ◦ DO-178B does a good job ensuring the software correctly implements its low and high level specifications
  ◦ Faults are likely to be in these specifications
  Need higher-level, independent specifications
Safety Cases and Formal Monitors

• Intellectual basis for assurance in support of certification is a credible argument based on documented evidence that supports suitable claims

• DO-178B is an example of standards-based assurance
  ○ Specifies just the evidence to be developed
  ○ The claims and argument are largely implicit

  Effective in slow-moving fields, but can be a barrier and a hazard to innovation

• Hence, growing interest in safety-case approach to assurance
  ○ Make all of the argument, claims, evidence explicit

• Aha: monitor against the (sub)claims in the safety case

• Formal monitors are synthesized from or verified against safety claims using automated formal methods
Interpretation for Formal Monitors

- In a monitored architecture
  - Have an **operational** channel $A$ completely responsible for functions of the system
  - And a **monitor** $B$ that can trigger an alarm if it sees violation of safety properties
  - Requires higher level fault-recovery
  - So really an **subsystem** architecture
- Reuse previous analysis, where $A$ has **only** failures of omission
- **Demands** arrive at some constant rate per unit time
- **Non-demands** arrive each time $A$ succeeds
- Hence,

\[
\text{risk/unit time} \leq c_1 \times (C + P_{A1} \times P_{B1}) + (1 - P_{A1}) \times c_2 \times P_{B2}
\]

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Consequences For Formal Monitors

• Our analysis yields prob. of failure wrt. failures of omission in monitored system as \((C + P_{A1} \times P_{B1})\), vs. \(P_{A1}\) without monitor

• Credible and modest claims for perfection of a monitor (e.g., \(P_{B1} < 10^{-3}\)) deliver useful improvement

• Provided probability of common cause faults \(C\) is small

• I think it can be, because the monitor is derived from the safety case
Consequences For Formal Monitors (ctd.)

- But we also need to be concerned about failures of **commission**: risk is $c_2 \times P_{B_2}$

- These depend on the monitor **alone**

- **Cost** of these failures must be commensurate with credible claims for probability of perfection
  - A340 fuel system monitor: warn pilot—**OK**
  - A300 roll rate anomaly: reboot EFIS bus—**not OK**

- Imperfection wrt. failures of commission likely depends more on **selection of monitored properties** than correctness of the monitor

- Hence, selection of these properties is **critical**
Summary

- Started with analysis of 1002 systems
  - Failure of one channel and imperfection of the other are conditionally independent at the aleatory level
  - Only dependence is in epistemic assessment of their probabilities
  - Dependencies can be absorbed in a common-cause probability $C$
- The analysis was extended to failures of commission
- Then carried over to monitored systems
- And the epistemic failure rates and risk depend on $C, P_{A1}, P_{B1}, P_{B2}$ and $f, c_1, c_{B2}$
- It is feasible to assess these parameters
Conclusions

- **Asymmetric 1oo2 systems**, and **monitored systems** are plausible ways to **achieve** high reliability.

- With a **possibly perfect** channel they also provide a credible way to **assess** it.

- Risk of **failures of commission** (false alarms) requires careful consideration and engineering: for formal monitors, focus should be on **choice of monitored properties**.

- Reasonable rates of perfection require only **modest guarantees for the prover**; suggested how these can be provided without compromising performance.

- Caution: focus was on failure of monitored **sub**systems—we still have to respond to those failures at the system level.
Research Topics

• Can significant properties be monitored at the subsystem level, or are they emergent?

• More generally, can we develop approaches to assurance cases that are compositional?
  ○ Given the cases for components
  ○ Assemble these to provide case for system
  ○ Or for new context of deployment

These are very difficult topics (cf. IMA)

• We have a plausible approach for NSA-grade security
  ○ The MILS approach

• Yet more generally, can we assess assurance cases reliably?
  ○ Currently, it’s all human judgement
  ○ Reserve this for where it’s really indispensable
  ○ Formalize and automate all that can be